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## Majorana Neutrinos, Neutrino Mass Spectrum, CP-Violation and Neutrinoless Double $\beta$ -Decay: II. Mixing of Four Neutrinos

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### Abstract

Assuming four-neutrino mixing and massive Majorana neutrinos, we study the implications of the neutrino oscillation solutions of the solar and atmospheric neutrino problems, of the results of the LSND experiment and of the constraints on neutrino oscillations, obtained in reactor and accelerator experiments, for the predictions of the effective Majorana mass in neutrinoless double beta  $((\beta\beta)_{0\nu})$ -decay,  $|\langle m \rangle|$ . All 4-neutrino mass spectra compatible with the existing neutrino mass and oscillation data are considered: 2+2A,B and 3+1A,B,C. The general case of CP-nonconservation is investigated. The predicted values of  $|\langle m \rangle|$  depend strongly on the value of the lightest neutrino mass  $m_1$ , on the type of the neutrino mass spectrum, on the LSND neutrino mass-squared difference  $\Delta m_{\text{SBL}}^2$ , on the solution of the solar neutrino problem, as well as on the values of the three Majorana CP-violating phases, present in the lepton mixing matrix. If CP-invariance holds,  $|\langle m \rangle|$  is very sensitive to the values of the relative CP-parities of the massive Majorana neutrinos. We have also analyzed in detail the question of whether a measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV in the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments (NEMO3, CUORE, EXO, GENIUS), combined with the data from the solar, atmospheric, reactor and accelerator neutrino oscillation experiments and from the future neutrino mass  $^3\text{H}$   $\beta$ -decay experiment KATRIN would allow, and under what conditions, i) to determine the absolute values of the neutrino masses and thus the neutrino mass spectrum, and ii) to establish the existence of CP-violation in the lepton sector. We have pointed out, in particular, that the 2+2A and 3+1A spectra can be critically tested by the KATRIN experiment. The latter, in particular, can provide information on the value of the lightest neutrino mass,  $m_1$ , in the cases of the spectra 2+2A, 3+1A, 3+1B and 3+1C. For these neutrino mass spectra there exists a direct relation between  $|\langle m \rangle|$  or  $m_1$  and the neutrino mass measured in  $^3\text{H}$   $\beta$ -decay,  $m_{\nu_e}$ , and the measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV and of  $m_{\nu_e} \gtrsim 0.4$  eV will give the unique possibility to determine the absolute values of all four neutrino masses and to obtain information on CP-violation in the lepton sector.

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# 1 Introduction

With the accumulation of more and stronger evidences for oscillations of the atmospheric ( $\nu_\mu$ ,  $\bar{\nu}_\mu$ ) [1, 2] and solar ( $\nu_e$ ) [3, 4, 5, 6, 7, 8, 9, 10, 11] neutrinos, caused by neutrino mixing and nonzero neutrino masses in vacuum (see, e.g., [12, 13, 14, 15]), the problem of the nature of massive neutrinos emerges as one of the fundamental problems in the studies of neutrino mixing. Massive neutrinos, as is well known, can be Dirac or Majorana particles. In the former case they possess a conserved lepton charge and distinctive antiparticles, while in the latter there is no conserved lepton charge and massive neutrinos are truly neutral particles identical with their antiparticles (see, e.g., [13]). Massive Dirac neutrinos and neutrino mixing arise in gauge theories in which the individual lepton charges,  $L_e$ ,  $L_\mu$  and  $L_\tau$ , are not conserved, but a specific combination of the latter, which could be the total lepton charge  $L = L_e + L_\mu + L_\tau$ , or, e.g., the charge [16]  $L' = L_e - L_\mu + L_\tau$ , is conserved. Massive Majorana neutrinos arise if no lepton charge is conserved by the interactions responsible for the neutrino mass generation. Thus, the question of the nature of massive neutrinos is directly related to the question of the basic symmetries of the fundamental particle interactions.

The Majorana nature of massive neutrinos can be revealed by investigation of processes in which the total lepton charge  $L$  is not conserved and changes by two units,  $\Delta L = 2$ . The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron) is the neutrinoless double  $\beta$  ( $(\beta\beta)_{0\nu}$ ) decay of certain even-even nuclei (see, e.g., [17, 18, 13]):

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-. \quad (1)$$

If the  $(\beta\beta)_{0\nu}$ -decay is generated *only by the (V-A) charged current weak interaction through the exchange of virtual massive Majorana neutrinos*, the probability amplitude of this process is proportional in the case of neutrinos having masses not exceeding few MeV to the so-called “effective Majorana mass parameter”

$$\langle m \rangle \equiv \sum_{j=1} U_{ej}^2 m_j, \quad (2)$$

where  $m_j$  is the mass of the Majorana neutrino  $\nu_j$  and  $U_{ej}$  is the element of neutrino (lepton) mixing matrix.

Assuming mixing of three massive Majorana neutrinos, we have studied in [19] the implications of the solar and atmospheric neutrino oscillation data and of the data of the long baseline reactor experiments CHOOZ and Palo Verde for the predictions of the effective Majorana mass  $|\langle m \rangle|$ , which determines <sup>3</sup> the rate of the  $(\beta\beta)_{0\nu}$ -decay. Our study was stimulated, in part, by the expected future progress in the experimental searches for  $(\beta\beta)_{0\nu}$ -decay. The presently existing most stringent constraint on the value of the effective Majorana mass parameter was obtained in the <sup>76</sup>Ge Heidelberg-Moscow experiment [29]:

$$|\langle m \rangle| < 0.35 \text{ eV}, \quad 90\% \text{ C.L.} \quad (3)$$

Taking into account a factor of 3 uncertainty associated with the calculation of the relevant nuclear matrix element (see, e.g., [17, 18]) one finds

$$|\langle m \rangle| < (0.35 \div 1.05) \text{ eV}, \quad 90\% \text{ C.L.} \quad (4)$$

The IGEX collaboration has obtained [30]:

$$|\langle m \rangle| < (0.33 \div 1.35) \text{ eV}, \quad 90\% \text{ C.L.} \quad (5)$$

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<sup>3</sup>There exists an extensive literature on the predictions for  $|\langle m \rangle|$  in the case of three-neutrino mixing and massive Majorana neutrinos: see, e.g., refs. [20, 21, 22, 23, 24, 25, 26, 27, 28].

A sensitivity to  $|\langle m \rangle| \sim 0.10$  eV is foreseen to be reached in the currently operating NEMO3 experiment [31], while the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments CUORE, EXO, GENIUS [32, 33, 34], is planned to reach a sensitivity to values of  $|\langle m \rangle| \gtrsim 0.01$  eV, which are considerably smaller than the presently existing most stringent upper bounds (4) and (5).

We have found in [19], in particular, that the observation of the  $(\beta\beta)_{0\nu}$ -decay with a rate corresponding to  $|\langle m \rangle| \gtrsim 0.02$  eV, which is in the range of sensitivity of the future  $(\beta\beta)_{0\nu}$ -decay experiments, will provide unique information not only on the nature of massive neutrinos, but also on the neutrino mass spectrum. Combined with information on the value of the lightest neutrino mass or on the type of the neutrino mass spectrum, it can provide also information on the CP-violation in the lepton sector, and if CP-invariance holds - on the relative CP-parities of the massive Majorana neutrinos. A measured value of  $|\langle m \rangle| \gtrsim (2-3) \times 10^{-2}$  eV, for instance, would strongly disfavor (if not rule out), under the general assumptions of our study (3-neutrino mixing,  $(\beta\beta)_{0\nu}$ -decay generated only by the charged (V-A) current weak interaction via the exchange of the three Majorana neutrinos, neutrino oscillation solutions of the solar neutrino problem and atmospheric neutrino anomaly) the possibility of a hierarchical neutrino mass spectrum, while a value of  $|\langle m \rangle| \gtrsim (2-3) \times 10^{-1}$  eV would rule out the hierarchical neutrino mass spectrum, strongly disfavor the spectrum with inverted mass hierarchy and favor the quasi-degenerate spectrum.

In the present article we present predictions for  $|\langle m \rangle|$  in the case of mixing of four massive Majorana neutrinos. The assumption of mixing of four massive neutrinos allows to explain by neutrino oscillations both the results of the solar and atmospheric neutrino experiments, which requires two different neutrino mass-squared differences, and the indications for  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  oscillations reported by the LSND collaboration [35], which requires an additional much larger neutrino mass-squared difference. The fourth weak-eigenstate neutrino involved in the mixing must be (see, e.g., [15]) a sterile neutrino,  $\nu_s$  [36]. The latter can be accommodated, e.g., in extensions of the Standard Theory, which include  $SU(2)_L$  singlet right-handed (RH) neutrino field(s) (see, e.g. [37]). More specifically, we study in detail the implications of the neutrino oscillation solutions of the solar and atmospheric neutrino problems, of the results of the analysis of the LSND data and of the constraints on neutrino oscillations, obtained in reactor and accelerator experiments, for the predictions of the effective Majorana mass,  $|\langle m \rangle|$ . The effective Majorana mass  $|\langle m \rangle|$  in schemes with 4-neutrino mixing was discussed earlier in, e.g., [24, 38, 27, 39, 40]. We consider five different types of neutrino mass spectrum, compatible with the data on neutrino oscillations: using standard notations we denote them as 2+2A, 2+2B, 3+1A, 3+1B and 3+1C. These are essentially all possible types of 4-neutrino mass spectra compatible with the existing data. The general case of CP-nonconservation in the lepton sector is investigated, although we study the possibility of CP-invariance as well. In the latter case we obtain predictions for  $|\langle m \rangle|$  for all different sets of values of the relative CP-parities of the massive Majorana neutrinos. In both cases we pay special attention to the possibility of cancellations between the contributions to  $|\langle m \rangle|$  of the different massive Majorana neutrinos. We give, in particular, detailed predictions for the value of  $|\langle m \rangle|$  for the five different types of neutrino mass spectrum indicated above. We analyze also in detail the question of whether a measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV in the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments, combined with the data from the solar, atmospheric, reactor and accelerator neutrino oscillation experiments and from the future neutrino mass  $^3\text{H}$   $\beta$ -decay experiments (KATRIN, etc.) would allow, and under what conditions, i) to determine the absolute values of the neutrino masses and thus the neutrino mass spectrum, and ii) to establish the existence of CP-violation in the lepton sector.

## 2 Experimental Data and Constraints on Neutrino Masses and Mixing Angles

We present in this Section the constraints on the neutrino mass squared differences, neutrino (lepton) mixing angles and on the neutrino masses, which follow from the existing experimental data and will be used in our further analyzes.

Strong evidences for neutrino oscillations have been obtained in the experiments with atmospheric [1, 2] and solar [3, 4, 5, 6, 7, 8, 9, 10, 11] neutrinos. Indications for  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  oscillations were reported by the accelerator LSND collaboration [35]. The interpretation of these results in terms of neutrino oscillations requires three independent neutrino mass-squared differences,  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_\odot^2$ ,  $\Delta m_{\text{SBL}}^2$ , respectively, and thus (at least) 4-neutrino mixing. The fourth (weak-eigenstate) neutrino participating in the mixing together with the three flavour neutrinos,  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$ , must be a sterile neutrino,  $\nu_s$  (see, e.g., [15, 37]). The combined neutrino oscillation analyzes of the atmospheric, solar and LSND data have been performed so far assuming that  $\Delta m_{\text{SBL}}^2$ ,  $\Delta m_{\text{atm}}^2$  and  $\Delta m_\odot^2$  obey the relation (see, e.g., [42, 41]):

$$|\Delta m_\odot^2| \ll |\Delta m_{\text{atm}}^2| \ll |\Delta m_{\text{SBL}}^2|. \quad (6)$$

This hierarchical relation is suggested by the results of the two-neutrino oscillation analyzes of the atmospheric, solar and LSND data. The predictions for the effective Majorana mass,  $|<m>|$ , we are interested in, depend, in general, on the values of  $|\Delta m_{\text{SBL}}^2|$ ,  $|\Delta m_{\text{atm}}^2|$  and  $|\Delta m_\odot^2|$ , on the mixing angles which determine the solar neutrino oscillations/transitions, as well as on the limits on the neutrino oscillation parameters obtained in the reactor  $\bar{\nu}_e$  disappearance experiments BUGEY, CHOOZ and Palo Verde.

There are two general types of 4-neutrino mass spectra compatible with the existing data on neutrino masses and neutrino oscillations: i) the 2+2 one which is characterized by two light neutrinos and two quasi-degenerate heavier ones, the two corresponding pairs of neutrino mass-squared differences being separated by the LSND “gap”  $\sim \Delta m_{\text{SBL}}^2$ , and ii) the 3+1 type consisting of three neutrinos which are quasi-degenerate in mass and of one much lighter or much heavier neutrino separated (in mass-squared) from the other three by the LSND “gap”  $\sim \Delta m_{\text{SBL}}^2$ .

In the case of 3+1 type of neutrino mass spectrum, the negative results of reactor and accelerator short baseline disappearance neutrino oscillation experiments imply a strong upper bound on the amplitude of the  $\nu_\mu \rightarrow \nu_e$  transition,  $A_{\mu;e}$ . Comparing this bound with the allowed values of  $A_{\mu;e}$ , obtained in the analysis of the LSND data, lead to the conclusion [43, 44, 45] that the 3+1 schemes are disfavored by the data. At the same time it was shown [43, 44, 45] that the 2+2 schemes are fully compatible with all existing neutrino oscillation data. Recently, as a result of a new LSND data analysis, the region of allowed values of  $A_{\mu;e}$  changed and it was demonstrated [46, 47, 48] that the 3+1 schemes are compatible with all existing data for some specific values of  $\Delta m_{\text{SBL}}^2$ . This problem was further investigated in [49]. It was shown by a comprehensive statistical analysis that in the case of 3+1 schemes there is no overlapping of the region limited by the 95% C.L. upper bound on  $A_{\mu;e}$  and of the 99% C.L. LSND allowed region. All existing neutrino oscillation data can be described at 99% C.L. if  $\Delta m_{\text{SBL}}^2$  has values in limited regions around the points  $\sim 6 \text{ eV}^2$ ,  $1.7 \text{ eV}^2$ ,  $0.9 \text{ eV}^2$ ,  $0.3 \text{ eV}^2$ . Thus, the 3+1 schemes cannot be completely excluded by the analysis of the existing data. Taking this into account we will consider in this paper both the 2+2 and the 3+1 types of 4-neutrino mass spectra.

The results of the combined analysis of the solar and atmospheric neutrino data depend on the spectrum considered. At the same time, for the indicated two types of 4-neutrino mass spectra, the results of the 2-neutrino mixing analysis of the LSND data, which are given below (see eqs. (7) and (8)), can be used to constrain the elements of the neutrino mixing matrix in the 4-neutrino mixing case.

Let us discuss next briefly the LSND result. Searching for  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  oscillations with a high intensity source of  $\nu_\mu$ ,  $\bar{\nu}_\mu$  and  $\nu_e$  in the range of energies  $E \sim (30 - 60)$  MeV and detector located at a distance  $L \sim 30$  m from the neutrino source, the LSND collaboration observed an excess of  $e^+$ -like events over the background [35]. This excess was interpreted as being due to the  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  transitions. The search for the same transitions under somewhat less favorable experimental conditions (neutrino beam of smaller intensity,  $L \sim 18$  m) by the KARMEN experiment gave negative results [50]. Performing a joint analysis of the LSND and KARMEN data and taking into account the limits from the BUGEY reactor antineutrino experiment [51], one finds (at 95% C.L.) the following allowed region of values of the corresponding two-neutrino oscillation parameters,  $\Delta m_{\text{SBL}}^2$  and  $\sin^2 \theta_{\text{SBL}}$ , for which the LSND excess of events can be explained by  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  oscillations:

$$1.6 \times 10^{-1} \text{ eV}^2 \leq \Delta m_{\text{SBL}}^2 \leq 2.0 \text{ eV}^2 \quad (95\% \text{ C.L.}), \quad (7)$$

$$8.0 \times 10^{-4} \leq \sin^2 \theta_{\text{SBL}} \leq 4 \times 10^{-2} \quad (95\% \text{ C.L.}). \quad (8)$$

The upper and lower limits in (7) and (8) are strongly correlated; in particular, both  $\Delta m_{\text{SBL}}^2|_{\text{MAX}}$  and  $\Delta m_{\text{SBL}}^2|_{\text{MIN}}$  are decreasing functions of  $\sin^2 \theta_{\text{SBL}}$ . This inter-dependence of the allowed values of  $\Delta m_{\text{SBL}}^2$  and  $\sin^2 \theta_{\text{SBL}}$  has been taken into account in our analysis. Note also that the sign of  $\Delta m_{\text{SBL}}^2$  cannot be determined from the analysis of the data. We have assumed above for concreteness that  $\Delta m_{\text{SBL}}^2 > 0$ . Let us note that the LSND results on  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  oscillations will be tested in the accelerator experiment MiniBooNE [52] which is under preparation.

Consider next the results of the analysis of the solar and atmospheric neutrino oscillation data. We will discuss first the case of neutrino mass spectrum of the 2+2 type. The Super-Kamiokande atmospheric neutrino data [1, 2] is best interpreted in terms of dominant  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  oscillations [2]; some non-dominant fraction of the atmospheric  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) can oscillate into  $\nu_s$  and  $\nu_e$  ( $\bar{\nu}_e$ ). The  $\nu_\mu \rightarrow \nu_{\tau,s}$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_{\tau,s}$  oscillations of the atmospheric neutrinos of interest are characterized by three parameters (see, e.g., [42, 41]):  $\Delta m_{\text{atm}}^2$ ,  $\theta_{\text{atm}}$  and  $\cos^2 \beta$ , where  $\beta$  is a neutrino mixing angle. Of these three parameters only  $\Delta m_{\text{atm}}^2$  enters into the expression for  $|\langle m \rangle|$ . Under the condition (6), the oscillations/transitions of solar neutrinos are characterized by three parameters [42] as well:  $\Delta m_\odot^2 > 0$ ,  $\theta_\odot$  (or  $\tan^2 \theta_\odot$ ) and  $\cos^2 \beta$ . As we have indicated, the probability of the atmospheric  $\nu_\mu \rightarrow \nu_{\tau,s}$  ( $\bar{\nu}_\mu \rightarrow \bar{\nu}_{\tau,s}$ ) oscillations depends also on  $\cos^2 \beta$ . The parameter  $\cos^2 \beta$  determines, in particular, what fraction of the solar  $\nu_e$  and of the atmospheric  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) can transform into a sterile neutrino  $\nu_s$  ( $\bar{\nu}_s$ ). For, e.g.,  $\cos^2 \beta = 0$ , the solar  $\nu_e \rightarrow \nu_s$  transitions do not take place at all, while the atmospheric  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) can oscillate *only* into  $\nu_s$  ( $\bar{\nu}_s$ )<sup>4</sup>. If  $\cos^2 \beta = 1.0$ , the solar  $\nu_e$  can undergo transitions *only* into  $\nu_s$ , while the atmospheric  $\nu_\mu$  ( $\bar{\nu}_\mu$ ) *cannot undergo transitions into*  $\nu_s$  ( $\bar{\nu}_s$ ).

For the 2+2 type neutrino mass spectrum and the hierarchy (6) of neutrino mass-squared differences, a 4-neutrino oscillation analysis of the most recent solar neutrino data, including the new high precision Super-Kamiokande results [8] on the spectrum of the recoil electrons and on the day-night (D-N) effect, was performed in ref. [42]. The regions of neutrino oscillation parameters, corresponding to the large mixing angle (LMA) MSW, small mixing angle (SMA) MSW (see, e.g., [14, 15]), of the LOW and the quasi-vacuum oscillation (QVO) solutions of the solar neutrino problem (see, e.g., [53, 54, 55, 56]), allowed by the data at a given C.L. were determined. The existence of a given solution and the corresponding solution regions were shown to depend on the value of  $\cos^2 \beta$ . For  $\cos^2 \beta = 0$ , one recovers the results derived in the case of the two-neutrino  $\nu_e \rightarrow \nu_{\mu(\tau)}$  oscillations analyzes of the solar neutrino data (see, e.g., [53, 54, 56]). If, however,  $\cos^2 \beta = 1$ , only the  $\nu_e \rightarrow \nu_s$  transitions of solar neutrinos are possible and one finds that only the MSW SMA  $\nu_e \rightarrow \nu_s$  transition solution is allowed by the data [42]. As  $\cos^2 \beta$  changes from 0 to 1, the LMA and LOW-QVO solution regions diminish and disappear, while the SMA MSW solution region essentially changes only its position with respect to the  $\Delta m_\odot^2$ -axis, moving as a whole to

<sup>4</sup>This case is strongly disfavored (if not excluded) by the atmospheric neutrino data [2, 41].

smaller by a factor of  $\sim 1.2$  values of  $\Delta m_{\odot}^2$  [57]. For, e.g.,  $\cos^2 \beta = 0.30$ , the allowed regions of the LMA and LOW-QVO solutions are somewhat smaller than in the case of  $\cos^2 \beta = 0$  [42]. Both the LMA and LOW-QVO solution regions include  $\cos 2\theta_{\odot} = 0$  at 99% C.L., as like the analogous  $\cos^2 \beta = 0$  solution regions. The maximal values of  $\cos^2 \beta$ , for which one still has LMA, LOW and QVO solutions (at 99% C.L.) are [42]  $\cos^2 \beta \cong 0.72$ ; 0.77; 0.80. The  $\chi^2$ -analysis performed in [42] showed also that as  $\cos^2 \beta$  increases from 0 to 1, the quality of the fit of the data for the LMA and LOW-QVO solutions decreases, while that provided by the SMA MSW solution remains essentially unchanged. It should be added that the SMA MSW solution is excluded at 68% C.L., but becomes allowed approximately at 90% C.L. [42].

A neutrino oscillation analysis of the Super-Kamiokande atmospheric neutrino data in the case of four-neutrino mixing and 2+2 type of neutrino mass spectrum was performed in [41]. The analysis showed, in particular, that the atmospheric neutrino data excludes the possibility of  $\cos^2 \beta = 0$ : one finds that  $\cos^2 \beta \geq 0.33$  at 90% C.L. [41]. In the case of  $\cos^2 \beta = 0.4$ , for instance, we have  $\nu_{\mu} \rightarrow \nu_{\tau,s}$  and  $\nu_e \rightarrow \nu_{s,\tau}$  transitions respectively of the atmospheric and solar neutrinos. For  $\cos^2 \beta = 0.30$ ; 0.50, the allowed values of  $\Delta m_{\text{atm}}^2$  at 90% C.L. (99% C.L.) are the following [41]:

$$\text{for } \cos^2 \beta = 0.3 : \quad 2.0 \text{ (1.5)} \times 10^{-3} \text{ eV}^2 \leq \Delta m_{\text{atm}}^2 \leq 5.0 \text{ (6.5)} \times 10^{-3} \text{ eV}^2, \quad (9)$$

$$\text{for } \cos^2 \beta = 0.5 : \quad 2.5 \text{ (2.0)} \times 10^{-3} \text{ eV}^2 \leq \Delta m_{\text{atm}}^2 \leq 4.5 \text{ (6.5)} \times 10^{-3} \text{ eV}^2. \quad (10)$$

The sign of  $\Delta m_{\text{atm}}^2$  is undetermined by the data. The best fit value of  $|\Delta m_{\text{atm}}^2|$  found in [41] is:  $|\Delta m_{\text{atm}}^2|_{\text{BF}} = 3.2 \times 10^{-3} \text{ eV}^2$ .

It should be obvious from the preceding discussion that there exists a correlation between the allowed values of  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_{\odot}^2$ ,  $\tan^2 \theta_{\odot}$  and of  $\cos^2 \beta$ . Thus, although  $\cos^2 \beta$  does not enter explicitly into the expression for  $|<m>|$ , the latter depends implicitly on  $\cos^2 \beta$  through the allowed values of  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_{\odot}^2$  and  $\tan^2 \theta_{\odot}$ , which depend on the value of  $\cos^2 \beta$ . In what follows (Sections 4 and 5) we will present results for  $|<m>|$  in the case of the 2+2 type of neutrino mass spectrum for two values of  $\cos^2 \beta = 0.3$ ; 0.5. The corresponding values of  $\Delta m_{\text{atm}}^2$ , and of  $\Delta m_{\odot}^2$  and  $\tan^2 \theta_{\odot}$ , are given in eqs. (9)-(10) and in Table 1, respectively. The predictions for  $|<m>|$  for  $\cos^2 \beta \gtrsim 0.8$  practically coincide with those obtained for  $\cos^2 \beta = 0.3$  (0.5) in the case of the SMA MSW solution of the solar neutrino problem.

In the case of 3+1 type of neutrino mass spectrum, the results of the combined analysis of solar and atmospheric neutrino data reduce effectively to the results of the analysis performed under the assumption of 3-flavour neutrino mixing [55]. A detailed description of these 3-neutrino mixing results we will use in the corresponding analysis in Sections 6, 7 and 8 is given in [19] (see Section 2) and we are not going to reproduce them here.

It should be noted that given the results of the analysis of the atmospheric neutrino data, the inequality  $\Delta m_{\odot}^2 \ll \Delta m_{\text{atm}}^2$  holds for

$$\Delta m_{\odot}^2 \lesssim 2.0 \times 10^{-4} \text{ eV}^2. \quad (11)$$

This condition is fulfilled in the SMA MSW and LOW-QVO solutions regions and in part of the LMA MSW one [8, 42, 53, 54, 55]. According to ref. [42, 55], values of  $\Delta m_{\odot}^2 \sim (7.0-8.0) \times 10^{-3} \text{ eV}^2$  are allowed in the LMA MSW solution region. In this case the condition  $\Delta m_{\odot}^2 \ll \Delta m_{\text{atm}}^2$  is not satisfied and the reliability (accuracy) of the results derived is somewhat questionable. Thus, we will use in our further analysis the upper bound on  $\Delta m_{\odot}^2$  given in eq. (11), adding comments about how the results change if  $\Delta m_{\odot}^2 \sim 7.0 - 8.0 \times 10^{-3} \text{ eV}^2$ . This bound is also reported in Table 1.

Very important constraints on the oscillations of electron (anti-)neutrinos were obtained in the CHOOZ and Palo Verde disappearance experiments with reactor  $\bar{\nu}_e$  [58, 59]. For the 4-neutrino

Table 1: Values of  $\Delta m_{\odot}^2$  and  $\tan^2 \theta_{\odot}$  obtained in the analysis of the solar neutrino data in ref. [42] at 90% (99%) C.L. and used in the present study (see text for details). The best fit values (B.F.V.) are also given. The results shown correspond to  $\cos^2 \beta = 0$  (upper table),  $\cos^2 \beta = 0.3$  (middle table) and  $\cos^2 \beta = 0.5$  (lower table).

	$\Delta m_{\odot}^2$ [eV <sup>2</sup> ] [B.F.V.]	$\tan^2 \theta_{\odot}$ [B.F.V.]
LMA	$1.0 (1.0) \times 10^{-5} \div 2.0 (2.0) \times 10^{-4} [3 \times 10^{-5}]$	$0.2 (0.18) \div 0.8 (2.0) [0.35]$
SMA	$5.0 (4.0) \times 10^{-6} \div 6.0 (10) \times 10^{-6}$	$6.0 (2.0) \times 10^{-4} \div 1.0 (2.0) \times 10^{-3}$
LOW-QVO	$35 (0.5) \times 10^{-9} \div 2.0 (3.0) \times 10^{-7}$	$0.47 (0.4) \div 1.0 (3.0)$

	$\Delta m_{\odot}^2$ [eV <sup>2</sup> ]	$\tan^2 \theta_{\odot}$
LMA	$1.6 (1.2) \times 10^{-5} \div 0.8 (2.0) \times 10^{-4}$	$0.33 (0.25) \div 0.62 (1.1)$
SMA	$4.0 (4.0) \times 10^{-6} \div 7.5 (10) \times 10^{-6}$	$3.5 (2.0) \times 10^{-4} \div 1.0 (1.5) \times 10^{-3}$
LOW-QVO	$60 (0.6) \times 10^{-9} \div 6.0 (23) \times 10^{-8}$	$0.7 (0.45) \div 0.7 (2.8)$

	$\Delta m_{\odot}^2$ [eV <sup>2</sup> ]	$\tan^2 \theta_{\odot}$
LMA	$2.2 (1.6) \times 10^{-5} \div 2.2 (20) \times 10^{-5}$	$0.54 (0.35) \div 0.54 (0.9)$
SMA	$4.0 (4.0) \times 10^{-6} \div 6.5 (10) \times 10^{-6}$	$4.0 (2.0) \times 10^{-4} \div 0.9 (1.5) \times 10^{-3}$
LOW-QVO	$(6.0 \times 10^{-10} \div 1.5) \times 10^{-7}$	$(0.5 \div 2.2)$

mixing under discussion these constraints are relevant primarily in the case of neutrino mass spectrum of the 3+1 type. The two experiments were sensitive to values of  $\Delta m^2 \gtrsim 10^{-3}$  eV<sup>2</sup>, which includes the region of the solution of the atmospheric neutrino problem (see, e.g., eqs. (9) and (10)). No disappearance of the reactor  $\bar{\nu}_e$  was observed. Performing a two-neutrino oscillation analysis, the following rather stringent upper bound on the value of the corresponding mixing angle,  $\theta$ , was obtained <sup>5</sup> at 95% C.L. for  $\Delta m^2 \geq 1.5 \times 10^{-3}$  eV<sup>2</sup> [58]:

$$\sin^2 \theta < 0.09. \quad (12)$$

This limit holds in the case of the neutrino mass spectrum of the 2+2 type. For the 3+1 type of spectrum we will use the limit on  $\sin^2 \theta$ , obtained in the combined 3-neutrino mixing analysis of the solar, atmospheric and CHOOZ neutrino oscillation data in ref. [55]:  $\sin^2 \theta < 0.05$  (0.08) (90% (99%) C.L.). The precise upper limit on  $\sin^2 \theta$  following from the CHOOZ data is  $\Delta m^2$ -dependent [58]. This dependence is accounted for, whenever necessary, in our analysis. The sensitivity to the value of the parameter  $\sin^2 \theta$  is expected to be considerably improved by the MINOS experiment [60] in which the following upper limit can be reached:  $\sin^2 \theta < 5 \times 10^{-3}$ .

In the next few years the constraints on the values of  $\Delta m_{\text{SBL}}^2$ ,  $\sin^2 \theta_{\text{SBL}}$ ,  $\cos^2 \beta$ ,  $\Delta m_{\odot}^2$ ,  $\theta_{\odot}$ ,  $\Delta m_{\text{atm}}^2$ ,  $\theta_{\text{atm}}$  and  $\theta$  will be improved due to the increase of the statistics of the currently running experiments (e.g., SAGE [5], GNO [9], Super-Kamiokande [2, 8], SNO [10], K2K [61]) and the upgrade of some of them, as well as due to the data from the new experiments MiniBooNE [52], BOREXINO [62], KamLand [63], MINOS [60] and CNGS [64]. As a result, the values of the

<sup>5</sup>The possibility of large  $\sin^2 \theta > 0.9$  which is admitted by the CHOOZ data alone is incompatible with the neutrino oscillation interpretation of the solar neutrino deficit (see, e.g., [15, 25]).

neutrino oscillation parameters will be known with much better accuracy. For the 4-neutrino mixing schemes under study the results of the MiniBooNE experiment which is scheduled to begin data-taking in December of 2001, will be crucial: this experiment will test the LSND indications for  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations.

The Troitzk [65] and Mainz [66]  $^3\text{H}$   $\beta$ -decay experiments, studying the electron spectrum, provide information on the electron (anti-)neutrino mass  $m_{\nu_e}$ . The data contain features which are not well understood (a peak in the end-point region which varies with time [65]). The upper bounds given by the authors (at 95% C.L.) read:

$$m_{\nu_e} < 2.5 \text{ eV} \quad [65], \quad m_{\nu_e} < 2.9 \text{ eV} \quad [66]. \quad (13)$$

There are prospects to increase the sensitivity of the  $^3\text{H}$   $\beta$ -decay experiments and probe the region of values of  $m_{\nu_e} \sim (0.4 - 1.0) \text{ eV}$ : the next generation of  $^3\text{H}$   $\beta$ -decay experiment KATRIN [67] is planned to have a sensitivity to values of  $m_{\nu_e} \sim 0.35 \text{ eV}$ .

Cosmological and astrophysical data provide information on the sum of the neutrino masses. The current upper bound reads (see, e.g., [68] and the references quoted therein):

$$\sum_j m_j \lesssim 5.5 \text{ eV} . \quad (14)$$

The future experiments MAP and PLANCK can be sensitive to [69]

$$\sum_j m_j \cong 0.4 \text{ eV} . \quad (15)$$

In the next Sections we show that the data from the new generation of  $(\beta\beta)_{0\nu}$ -decay experiments, which will be sensitive to values of  $|<m>| \sim (0.01 - 0.10) \text{ eV}$ , can provide information on the neutrino mass spectrum as well as on the leptonic CP-violation generated by Majorana CP-violating phases.

### 3 Four-Neutrino Mixing and the $(\beta\beta)_{0\nu}$ -Decay Effective Majorana Mass

The explanation of the data of atmospheric neutrino, solar neutrino and LSND experiments in terms of neutrino oscillations requires the existence of mixing of (at least) four massive neutrinos. This means that in addition to the three active left-handed flavour neutrino fields  $\nu_{lL}$ ,  $l = e, \mu, \tau$ , which enter into the expression of the weak charged lepton current, one has to assume that there exists a sterile neutrino field,  $\nu_{sL}$  - a singlet of  $SU(2)_L \times U(1)_{Y_W}$ <sup>6</sup>, and that

$$\begin{aligned} \nu_{lL} &= \sum_{j=1}^4 U_{lj}^* \nu_{jL}, \quad l = e, \mu, \tau, \\ \nu_{sL} &= \sum_{j=1}^4 U_{sj}^* \nu_{jL}. \end{aligned} \quad (16)$$

Here  $\nu_{jL}$  is the left-handed field of the neutrino  $\nu_j$  having a mass  $m_j$  and  $U$  is a  $4 \times 4$  unitary mixing matrix. We will assume that the neutrinos  $\nu_j$  are Majorana particles whose fields satisfy the Majorana condition:

$$C(\bar{\nu}_j)^T = \nu_j, \quad j = 1, 2, 3, 4, \quad (17)$$

---

<sup>6</sup>The field  $\nu_{sL}$  is related to the "standard" RH  $SU(2)_L \times U(1)_{Y_W}$ -singlet neutrino field  $\nu_R$  through the charge conjugation matrix  $C$ :  $\nu_{sL} = C(\bar{\nu}_R)^T$  (see, e.g., [13]).



where  $C$  is the charge conjugation matrix. We will numerate the neutrino masses in such way that  $m_1 < m_2 < m_3 < m_4$ .

The effective Majorana mass parameter can be expressed as:

$$|<m>| \equiv |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 + m_4 U_{e4}^2|, \quad (18)$$

where  $U_{ei}$  are the entries of the  $U$  mixing matrix,  $m_i$  is the mass of the  $\nu_i$  massive neutrino. We have

$$U_{ej} = |U_{ej}| e^{i\alpha_j}, \quad (19)$$

where  $\alpha_j$ ,  $j = 1, 2, 3, 4$ , are real phases. Only the phase differences  $(\alpha_j - \alpha_k) \equiv \alpha_{jk}$  ( $j > k$ ) can play a physical role.

The Majorana condition can also have the form

$$C(\bar{\nu}_j)^T = (\xi_j^*)^2 \nu_j, \quad j = 1, 2, 3, 4, \quad (20)$$

where  $\xi_j$ ,  $j = 1, 2, 3, 4$ , are arbitrary phase factors. The effective Majorana mass parameter is given now by:

$$|<m>| \equiv |m_1 U_{e1}^2 \xi_1^2 + m_2 U_{e2}^2 \xi_2^2 + m_3 U_{e3}^2 \xi_3^2 + m_4 U_{e4}^2 \xi_4^2|. \quad (21)$$

Obviously, the physical Majorana CP-violating phases on which  $|<m>|$  depends are  $\alpha_{21} \equiv \arg(U_{e2}^2 \xi_2^2) - \arg(U_{e1}^2 \xi_1^2)$ ,  $\alpha_{31} \equiv \arg(U_{e3}^2 \xi_3^2) - \arg(U_{e1}^2 \xi_1^2)$  and  $\alpha_{41} \equiv \arg(U_{e4}^2 \xi_4^2) - \arg(U_{e1}^2 \xi_1^2)$ . All CP-violation effects associated with the Majorana nature of the massive neutrinos are generated by  $\alpha_{21} \neq k\pi$ ,  $\alpha_{31} \neq k'\pi$  and  $\alpha_{41} \neq k''\pi$ ,  $k, k', k'' = 0, 1, 2, \dots$

The  $4 \times 4$  mixing matrix  $U$  can be parametrized by 6 Euler-like angles and 10 phases. We can eliminate the unphysical phases in  $U$  by rephasing, e.g., the charged lepton,  $l(x)$ , and the neutrino,  $\nu_j(x)$ , fields in the weak charged lepton current,  $l(x) \rightarrow e^{im} l(x)$  and  $\nu_j(x) \rightarrow e^{i\beta_j} \nu_j(x)$ . The phases which re-appear in the Majorana condition for the fields of massive neutrinos when the latter are Majorana particles are physical [70, 71]: these phases will be present also in  $|<m>|$ <sup>7</sup>. The elements of the lepton mixing matrix and the phase factors in the Majorana condition for massive neutrino fields change as follows:

$$U_{lj} \rightarrow U_{lj} e^{-i(\eta - \beta_j)}, \quad l = e, \mu, \tau, \quad j = 1, 2, 3, 4, \quad (22)$$

$$\xi_j \rightarrow \xi_j e^{-i\beta_j}. \quad (23)$$

Thus, in the case of Dirac neutrinos there are four CP-violating phases in the neutrino (lepton) mixing matrix  $U$ , while in the case of massive Majorana neutrinos  $U$  contains seven CP-violating phases. However, since the sterile neutrino  $\nu_s$  does not participate in electroweak interactions<sup>8</sup> (see, e.g., refs. [13, 15, 37]), in both cases of massive Dirac and massive Majorana neutrinos one CP-violating phase does not appear in physical processes taking place at energy scales of the order of, or smaller than, the electroweak symmetry breaking scale. This is the phase common for all elements of the fourth row of  $U$ , i.e., for the elements  $U_{sj}$ ,  $j = 1, 2, 3, 4$ , associated with the sterile neutrino field  $\nu_{sL}$ . Since the field  $\nu_{sL}$  does not enter into the charged current and neutral current weak interaction Lagrangian  $\mathcal{L}^{CC+NC}$ , the indicated phase will not appear in processes generated by  $\mathcal{L}^{CC+NC}$ . Thus, there exist at most six *relevant* physical CP-violating phases in the lepton sector

<sup>7</sup>In the case of massive Dirac neutrinos the indicated phases will just be eliminated from the neutrino mixing matrix  $U$  and from the charged current weak interaction Lagrangian by the indicated rephasing of the charged lepton and neutrino fields; thus, they are unphysical.

<sup>8</sup>We suppose that the sterile neutrino field  $\nu_{sL}$  is a singlet with respect to the electroweak gauge symmetry group, which is assumed to be that of the Standard Theory, i.e.,  $SU(2)_L \times U(1)_{Y_W}$ .

of the theory with massive Majorana neutrinos of interest, and consequently - at most six relevant rephasing invariants. Three are the standard Dirac ones,  $J_{1,2,3}$ , present in the case of mixing of four massive Dirac neutrinos [72, 73, 74] (see also [75]):

$$J_1 = \text{Im} (U_{\mu 3} U_{e 4} U_{\mu 4}^* U_{e 3}^*), \quad (24)$$

$$J_2 = \text{Im} (U_{\tau 3} U_{e 4} U_{\tau 4}^* U_{e 3}^*), \quad (25)$$

$$J_3 = \text{Im} (U_{\mu 2} U_{e 4} U_{\mu 4}^* U_{e 2}^*). \quad (26)$$

The existence of the other three,  $S_1$ ,  $S_2$  and  $S_3$ , is related to the Majorana nature of the massive neutrinos  $\nu_j$  (see [73, 74]):

$$S_1 \equiv \text{Im} (U_{e 1} U_{e 4}^* \xi_4^* \xi_1), \quad (27)$$

$$S_2 \equiv \text{Im} (U_{e 2} U_{e 4}^* \xi_4^* \xi_2), \quad (28)$$

$$S_3 \equiv \text{Im} (U_{e 3} U_{e 4}^* \xi_4^* \xi_3). \quad (29)$$

The Majorana CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$  are determined by the three independent rephasing invariants,  $S_{1,2,3}$ . We have, for instance:

$$\cos \alpha_{41} = 1 - 2 \frac{S_1^2}{|U_{e 1}|^2 |U_{e 4}|^2}, \quad (30)$$

$$\cos(\alpha_{41} - \alpha_{21}) = \cos(\alpha_4 - \alpha_2) = 1 - 2 \frac{S_2^2}{|U_{e 2}|^2 |U_{e 4}|^2}, \quad (31)$$

and

$$\cos(\alpha_{41} - \alpha_{31}) = \cos(\alpha_4 - \alpha_3) = 1 - 2 \frac{S_3^2}{|U_{e 3}|^2 |U_{e 4}|^2}. \quad (32)$$

One can express the Majorana CP-violating phases entering into the expression for  $|< m >|$  in terms of the rephasing invariants <sup>9</sup>  $S_{1,2,3}$ : the former are independent of the Dirac rephasing invariants  $J_{1,2,3}$ . This is a consequence of the specific choice of  $S_{1,2,3}$ , eqs. (27) - (29), which is not unique [73]. With this choice the amplitude of the  $K^+ \rightarrow \pi^- + \mu^+ + \mu^+$  decay, for instance, which, as like the  $(\beta\beta)_{0\nu}$ -decay, is generated by the exchange of the four virtual massive Majorana neutrinos in the scheme under discussion, depends, as can be shown, on all six rephasing invariants  $S_{1,2,3}$  and  $J_{1,2,3}$ .

If CP-invariance holds in the lepton sector we have, in particular,  $S_1 = 0$  or  $\text{Re} (U_{e 1} U_{e 4}^* \xi_4^* \xi_1) = 0$ ,  $S_2 = 0$  or  $\text{Re} (U_{e 2} U_{e 4}^* \xi_4^* \xi_2) = 0$ , and  $S_3 = 0$  or  $\text{Re} (U_{e 3} U_{e 4}^* \xi_4^* \xi_3) = 0$ . In terms of constraints on the phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$  this implies  $\alpha_{21} = k\pi$ ,  $\alpha_{31} = k'\pi$ ,  $\alpha_{41} = k''\pi$ ,  $k, k', k'' = 0, 1, 2, \dots$

In all our subsequent analyzes we will set for convenience (and without loss of generality)  $\xi_j = 1$ ,  $j=1,2,3,4$ , i.e., we will assume that the fields of the Majorana neutrinos  $\nu_j$  satisfy the Majorana conditions (17). The CP-invariance constraint on the elements of the lepton mixing matrix of interest reads <sup>10</sup> [77, 78, 79] (see also [13]):

$$U_{ej}^* = \eta_j^{CP} U_{ej}, \quad (33)$$

---

<sup>9</sup>Let us note that the Majorana CP-violating phases of interest do not enter into the expressions for the  $\nu_l \rightarrow \nu_{l'}$  ( $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ ) and  $\nu_{l(s)} \rightarrow \nu_{s(l)}$  ( $\bar{\nu}_{l(s)} \rightarrow \bar{\nu}_{s(l)}$ ),  $l, l' = e, \mu, \tau$ , oscillation/transition probabilities [70, 76].

<sup>10</sup>This constraint is obtained from the requirement of CP-invariance of the charged current weak interaction Lagrangian  $\mathcal{L}^{CC}$ , by choosing the arbitrary phase factors in the CP-transformation laws of the electron and the  $W$ -boson fields equal to 1.

where  $\eta_j^{CP} = i\phi_j = \pm i$  is the CP-parity of the Majorana neutrino  $\nu_j$  with mass  $m_j > 0$ . In this case  $|\langle m \rangle|$  is given by:

$$|\langle m \rangle| \equiv \left| \sum_{j=1}^4 \eta_j^{CP} |U_{ej}|^2 m_j \right| = \left| \sum_{j=1}^4 \phi_j |U_{ej}|^2 m_j \right|. \quad (34)$$

The neutrino oscillation experiments provide information on  $\Delta m_{jk}^2 = m_j^2 - m_k^2$  ( $j > k$ ). In the case of 4-neutrino mixing (16) there are three independent  $\Delta m^2$  parameters. The four neutrino masses  $m_j$ ,  $j = 1, 2, 3, 4$ , can be expressed in terms of these three parameters and, e.g., of  $m_1$ . We have:

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad (35)$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2}. \quad (36)$$

$$m_4 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{43}^2}. \quad (37)$$

The mass-squared difference inferred from the neutrino oscillation interpretation of the LSND data,  $\Delta m_{\text{SBL}}^2$ , is equal to  $\Delta m_{41}^2$ ,

$$\Delta m_{\text{SBL}}^2 = \Delta m_{41}^2 = \Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{43}^2, \quad (38)$$

while for the ones deduced from the solar and atmospheric neutrino data,  $\Delta m_{\odot}^2$  and  $\Delta m_{\text{atm}}^2$ , we have several possibilities:

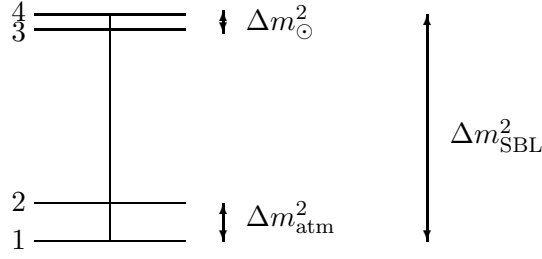
- in the case of the 2+2 type of neutrino mass spectrum we can identify two sub-cases: i) 2+2A - with  $\Delta m_{\odot}^2 = \Delta m_{43}^2$  and  $\Delta m_{\text{atm}}^2 = \Delta m_{21}^2$ , and ii) 2+2B - with  $\Delta m_{\odot}^2 = \Delta m_{21}^2$  and  $\Delta m_{\text{atm}}^2 = \Delta m_{43}^2$ ;
- for the 3+1A type of neutrino mass spectrum characterized by one neutrino being much lighter than the other three which are nearly degenerate, we have  $\Delta m_{\odot}^2 = \Delta m_{43}^2$ ,  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$ , or  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$ ,  $\Delta m_{\odot}^2 = \Delta m_{43}^2$ ;
- for the 3+1B and in the 3+1C neutrino mass spectra with one neutrino being much heavier than the other three, one finds, respectively,  $\Delta m_{\odot}^2 = \Delta m_{21}^2$ ,  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$ , and  $\Delta m_{\odot}^2 = \Delta m_{32}^2$ ,  $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2$ .

Evidently, getting information on the lightest neutrino mass  $m_1$  would be crucial for determining the values of  $m_{2,3,4}$ . The neutrino oscillation data and cosmological arguments (see, e.g., eq. (14)) suggest that  $m_1 \lesssim 1$  eV.

For each of the possible patterns of neutrino masses indicated above, we will study in detail in what follows the implications of the data on neutrino oscillations for the searches for  $(\beta\beta)_{0\nu}$ -decay.

## 4 The 2+2A Neutrino Mass Spectrum

The 2+2A neutrino mass spectrum is characterized by the following pattern of the neutrino masses  $m_j$ :



This pattern corresponds to the inequalities  $m_1 < m_2 < (\ll) m_3 \simeq m_4$ , or equivalently to:

$$\begin{aligned} m_1 &< (\ll) \sqrt{\Delta m_{41}^2}, \\ \sqrt{\Delta m_{43}^2} &\ll \sqrt{\Delta m_{21}^2} \ll \sqrt{\Delta m_{41}^2}. \end{aligned} \quad (39)$$

Taking into account these inequalities one can make the identification:

$$\Delta m_{21}^2 \equiv \Delta m_{\text{atm}}^2, \quad \Delta m_{43}^2 \equiv \Delta m_{\odot}^2, \quad \Delta m_{41}^2 \equiv \Delta m_{\text{SBL}}^2. \quad (40)$$

We will suppose that  $\Delta m_{\odot}^2$  takes values in the regions reported in Table 1 for  $\cos^2 \beta = 0.3; 0.5$ ,  $\Delta m_{\text{atm}}^2$  lies in the interval (9) or (10), and  $\Delta m_{\text{SBL}}^2$  can have values in the interval (7). From eqs. (35) - (37), (38) and (39) it follows that:

$$\begin{aligned} m_2 &= \sqrt{m_1^2 + \Delta m_{\text{atm}}^2}, \\ m_3 \simeq m_4 &= \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}. \end{aligned} \quad (41)$$

The condition  $m_1, m_2 \ll m_3, m_4$ , which could be valid for the neutrino mass spectrum under discussion, is satisfied for

$$m_1 < 0.25 \sqrt{\Delta m_{\text{SBL}}^2} \Rightarrow m_1 < 0.1 \text{ eV}. \quad (42)$$

For the elements  $U_{ej}$  of the neutrino mixing matrix  $U$  we have:

- i)  $|U_{e1}|^2$  and  $|U_{e2}|^2$  are constrained by the LSND results [35] and BUGEY neutrino oscillation limits [51] to lie in the interval:

$$2 \times 10^{-4} \leq |U_{e1}|^2 + |U_{e2}|^2 \leq 1 \times 10^{-2}; \quad (43)$$

- ii)  $|U_{e3}|$  and  $|U_{e4}|$  are related to the solar mixing angle  $\theta_{\odot}$ :

$$\begin{aligned} |U_{e3}|^2 &= \cos^2 \theta_{\odot} \left(1 - \sum_{i=1}^2 |U_{ei}|^2\right), \\ |U_{e4}|^2 &= \sin^2 \theta_{\odot} \left(1 - \sum_{i=1}^2 |U_{ei}|^2\right), \end{aligned} \quad (44)$$

where  $\theta_{\odot}$  takes values in the regions quoted in Table 1.

In the case of the 2+2A scheme under discussion, the effective neutrino mass  $m_{\nu_e}$ , which can be determined from the measurement of the end-point part of the  $\beta$ -spectrum of  ${}^3\text{H}$ , is given by

$$m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} \quad (45)$$

From the results of the analysis of the LSND data [35] it follows that for  $m_1 < 0.1$  eV one has:

$$0.4 \text{ eV} \leq m_{\nu_e} \leq 1.4 \text{ eV}. \quad (46)$$

If  $m_1 \gtrsim 0.1$  eV, the lower bound in (46), in particular, will be larger. Consequently, for the 2+2A type of neutrino mass spectrum and any value of  $m_1$ ,  $m_{\nu_e}$  is predicted to lie in the range planned to be probed by the future Karlsruhe-Mainz-Troitsk experiment KATRIN [67]. Thus, the realization of the KATRIN project will allow to check directly the possibility of 2+2A type of neutrino mass spectrum. A measurement of  $m_{\nu_e} \gtrsim 0.4$  eV and a more accurate knowledge of  $\Delta m_{\text{SBL}}^2$  would permit to determine the value of  $m_1$ . This would allow to determine also the values of  $m_{2,3,4}$  in the case of the 2+2A spectrum.

Using equation (18) and neglecting  $m_1|U_{e1}|^2$  and  $m_2|U_{e2}|^2$  with respect to the terms  $\sim m_3, m_4$ , we can express the effective mass  $|\langle m \rangle|$  in terms of  $m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$ ,  $\theta_\odot$  and the CP-violating phase difference  $(\alpha_4 - \alpha_3) = (\alpha_{41} - \alpha_{31})$  [24]:

$$|\langle m \rangle| \simeq m_{\nu_e} \sqrt{1 - \sin^2(2\theta_\odot) \sin^2 \frac{\alpha_{31} - \alpha_{41}}{2}}. \quad (47)$$

In eq. (47) we have neglected the contribution of the two lightest neutrinos since

$$|m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}}| \leq m_1|U_{e1}|^2 + m_2|U_{e2}|^2 \ll m_4 \quad (48)$$

For  $m_1$  satisfying eq. (42), this contribution does not exceed approximately  $1.5 \times 10^{-3}$  eV.

The effective Majorana mass  $|\langle m \rangle|$ , eq. (47), depends on the value of  $\theta_\odot$  and therefore the predictions for  $|\langle m \rangle|$  will vary with the solution of the solar neutrino problem.

Consider first the case of  $(\alpha_{41} - \alpha_{31}) \equiv \alpha_{43}$  taking CP-conserving values. There are two different possibilities.

**Case A.** If the neutrinos  $\nu_3$  and  $\nu_4$  have the same CP parities  $\phi_3 = \phi_4$  (i.e.  $\alpha_{41} = \alpha_{31} = 0, \pm\pi$ ), the expression for  $|\langle m \rangle|$  reduces to

$$|\langle m \rangle| \simeq m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}. \quad (49)$$

In this case  $|\langle m \rangle|$  coincides (up to corrections which do not exceed approximately  $10^{-2}$  eV) with the mass  $m_{\nu_e}$ , measured in  $^3\text{H}$   $\beta$ -decay experiments. Using the 95% C.L. results of the analysis [35] one finds that for any value of  $m_1$ ,  $|\langle m \rangle|$  should satisfy:

$$|\langle m \rangle| \geq 0.4 \text{ eV}. \quad (50)$$

If  $m_1$  is sufficiently small so that eq. (42) is fulfilled, one has also:  $|\langle m \rangle| \leq 1.4$  eV. Comparing the lower bound (50) with the upper bounds on  $|\langle m \rangle|$  given by eqs. (4) and (5) one can conclude that an improvement of these upper bounds by a factor  $\sim (3 - 4)$  would essentially rule out the possibility of equal CP-parities of  $\nu_3$  and  $\nu_4$  for the 2+2A neutrino mass spectrum.

**Case B.** If the neutrinos  $\nu_3$  and  $\nu_4$  have opposite CP parities  $\phi_3 = -\phi_4$  (i.e.  $\alpha_{41} = \alpha_{31} + \pi = 0, \pm\pi$ ), the effective Majorana mass  $|\langle m \rangle|$  is given by:

$$|\langle m \rangle| \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |\cos 2\theta_\odot|. \quad (51)$$

In this case  $|\langle m \rangle|$  depends on the value of  $\cos 2\theta_\odot$  and therefore on the particular solution of the solar neutrino problem considered. If the allowed region for  $\cos 2\theta_\odot$  includes  $\cos 2\theta_\odot = 0$ , for instance, there would be no significant lower bound on  $|\langle m \rangle|$ . More specifically, we have:

1. For the LMA MSW solution, the allowed values of  $\cos 2\theta_\odot$  depend on the value of  $\cos^2 \beta$ : the results of the analysis performed in [42] show that  $\cos 2\theta_\odot > 0$  both for  $\cos^2 \beta = 0.3$  at 90% C.L. and for  $\cos^2 \beta = 0.5$  at 90% and 99% C.L., while in the solution region found for  $\cos^2 \beta = 0.3$  at 99% C.L. one has  $|\cos 2\theta_\odot| \geq 0$ . Using the relevant 90% (99%) C.L. results for  $\theta_\odot$  one gets for  $m_1 < 0.1$  eV:

$$0.09 \text{ (0.0) eV} \leq |\langle m \rangle| \leq 0.7 \text{ (0.8) eV} , \quad \cos^2 \beta = 0.3; \quad (52)$$

$$0.13 \text{ (0.02) eV} \leq |\langle m \rangle| \leq 0.46 \text{ (0.67) eV} , \quad \cos^2 \beta = 0.5. \quad (53)$$

Note that the 90% C.L. allowed values of  $|\langle m \rangle|$  lie entirely in the region which will be probed in the currently running and next generation of  $(\beta\beta)_{0\nu}$ -decay experiments [31, 32, 34, 33]. For  $0.1 \text{ eV} \leq m_1 \lesssim 1.0 \text{ eV}$ , both the non-zero lower bounds and the upper bounds increase by factors 1.3 and 1.2, respectively. A better determination of the value of the solar neutrino mixing angle  $\theta_\odot$  would be very important for limiting further the range of possible values of  $|\langle m \rangle|$ .

2. For the SMA MSW solution we have  $\cos 2\theta_\odot \simeq 1$  and this case of CP-conservation cannot be distinguished from the earlier considered one,  $\phi_3 = \phi_4$ , or from the case of violation of the CP-symmetry due to  $\alpha_{43}$ : in all these cases we get

$$|\langle m \rangle| \simeq m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} \quad (54)$$

and, correspondingly, the same allowed values of  $|\langle m \rangle|$  as in eq. (50). Obviously, if the neutrino mass spectrum is of the 2+2A type and the SMA MSW solution (or Case A) is realized, the measurement of  $|\langle m \rangle| \gtrsim 0.4 \text{ eV}$ , or of  $m_{\nu_e} \gtrsim 0.4 \text{ eV}$ , and of  $\Delta m_{\text{SBL}}^2$  would allow to determine the lightest neutrino mass  $m_1$ , and, correspondingly, the neutrino mass spectrum.

3. For the LOW-QVO solution, the 90% C.L. and 99% C.L. results of the analysis of ref. [42] do not exclude the possibility of having  $|\langle m \rangle| \cong 0 \text{ eV}$ . Noting that for  $\cos^2 \beta = 0.5$ , the LOW-QVO solution of the solar neutrino problem is excluded at 90% C.L. and is allowed at 99% C.L., we find using the 90% (99%) C.L. allowed values of  $|\cos 2\theta_\odot|$  and 95% C.L. values of  $\Delta m_{\text{SBL}}^2$ :

$$0.08 \text{ (0.0) eV} \leq |\langle m \rangle| \leq 0.28 \text{ (0.66) eV} , \quad \cos^2 \beta = 0.3; \quad (55)$$

$$(0.02 \text{ eV}) \leq |\langle m \rangle| \leq (0.52 \text{ eV}) \quad \cos^2 \beta = 0.5. \quad (56)$$

These results correspond again to  $m_1 < 0.1 \text{ eV}$ . If  $0.1 \text{ eV} \leq m_1 \lesssim 1.0 \text{ eV}$ , both the non-zero lower bounds and the upper bounds increase by factors 1.3 and 1.2, respectively.

In Fig. 2 and in Fig. 3 we show the allowed values of  $|\langle m \rangle|$  for the LMA and LOW-QVO solutions of the solar neutrino problem, respectively, as a function of  $\sqrt{\Delta m_{\text{SBL}}^2}$  for the two cases of CP-conservation and in the case of CP-violation. The allowed region corresponding to CP-violation not only covers the two regions of CP-conservation, but extends also between the latter. Thus, there exists a region, marked by dark-grey color in each of the two figures, which can be spanned *only if the CP-parity is not conserved*, the “just-CP-violation” region: if the neutrino mass spectrum is of the 2+2A type, an experimental point in this region would signal CP-violation in the lepton sector. We have already noticed that for the SMA solution, the prediction for  $|\langle m \rangle|$  in the two CP-conserving cases and the CP-violating one practically coincide since  $\sin^2 2\theta_\odot < 10^{-2}$ .

For all solutions of the solar neutrino problem we have  $|\langle m \rangle| \leq 1.4 \text{ eV}$  if  $m_1 < 0.1 \text{ eV}$ . This maximal value is already excluded by the most stringent upper bounds [29, 30] on  $|\langle m \rangle|$ , eqs.

(4) and (5). In most of the allowed regions of the relevant parameter space one has  $|\langle m \rangle| \geq 0.01 \div 0.02$  eV, which can be tested in the  $(\beta\beta)_{0\nu}$ -decay experiments of the next generation.

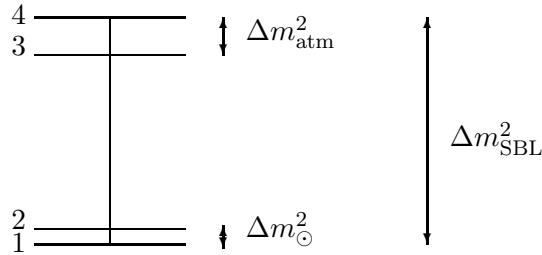
As the predictions for  $|\langle m \rangle|$  for the LMA and the LOW-QVO solutions and the 2+2A neutrino mass spectrum under study depend crucially on the value of  $\cos 2\theta_\odot$ , we show in Fig. 4  $|\langle m \rangle|$  as function of  $\cos 2\theta_\odot$ . The “just-CP-violation” region is marked by dark-grey color. The uncertainties in the predicted values of  $|\langle m \rangle|$  in the two cases of CP-conservation are mainly due to the uncertainty in the value of  $\Delta m_{\text{SBL}}^2$ . A better determination of  $\Delta m_{\text{SBL}}^2$  in the upcoming experiment MiniBooNE would lead to a “contraction” of the CP-conservation regions. Once  $\Delta m_{\text{SBL}}^2$  and  $\cos 2\theta_\odot$  are known with sufficient accuracy and if  $\cos 2\theta_\odot$  lies in the region of the LMA or LOW-QVO solution, it might be possible to establish whether the CP-symmetry is violated and to determine the value of one of the Majorana CP-violating phases,  $(\alpha_{41} - \alpha_{31})$  by measuring  $m_{\nu_e} \gtrsim 0.4$  eV, and  $|\langle m \rangle| \neq 0$ . Indeed, through equation (47), the value of the phase  $(\alpha_{41} - \alpha_{31})$  can be related to the experimentally measurable quantities  $|\langle m \rangle|^2$ ,  $m_{\nu_e}^2 \simeq (m_1^2 + \Delta m_{\text{SBL}}^2)$  and  $\theta_\odot$  [24]:

$$\sin^2 \frac{\alpha_{41} - \alpha_{31}}{2} \simeq \frac{1}{\sin^2 2\theta_\odot} \left( 1 - \frac{|\langle m \rangle|^2}{m_1^2 + \Delta m_{\text{SBL}}^2} \right) \simeq \frac{1}{\sin^2 2\theta_\odot} \left( 1 - \frac{|\langle m \rangle|^2}{m_{\nu_e}^2} \right). \quad (57)$$

In Fig. 5 we show the allowed ranges of  $\sin^2((\alpha_{41} - \alpha_{31})/2)$  as a function of  $|\langle m \rangle|$  in the case  $m_1^2 \ll \Delta m_{\text{SBL}}^2$ , for the LMA and LOW-QVO solutions of the solar neutrino problem (we remind the reader that in the case of the SMA MSW solution the value of  $|\langle m \rangle|$  practically does not depend on whether CP-invariance holds or not). For a given  $|\langle m \rangle|$ , the present uncertainties in the value of  $\sin^2(\alpha_{41} - \alpha_{31})/2$ , are determined by the uncertainties in the knowledge of  $\Delta m_{\text{SBL}}^2$  and  $\sin^2 2\theta_\odot$ . As the latter will diminish,  $\sin^2(\alpha_{41} - \alpha_{31})/2$ , will be restricted to lie in a smaller region and the measurement of  $|\langle m \rangle|$  can fix the value of  $\sin^2(\alpha_{41} - \alpha_{31})/2$  with some uncertainty. Obviously, this would be possible if the neutrino mass spectrum is of the 2+2A type. We would like to emphasize that in the case of the 2+2A spectrum, the Majorana CP-violating phases  $\alpha_{21}$  and  $\alpha_{31}$  cannot be constrained by the measurement of  $|\langle m \rangle| \gtrsim 10^{-2}$  eV. These two phases can be a source of CP-violation even if one finds that  $(\alpha_{41} - \alpha_{31}) = 0, \pm\pi$ .

## 5 The 2+2B neutrino mass spectrum

The 2+2B neutrino mass spectrum can be represented graphically as:



It can be defined through the inequalities  $m_1 \simeq m_2 < (\ll) m_3 < m_4$ . This spectrum can also be characterized by the following conditions:

$$m_1 < (\ll) \sqrt{\Delta m_{41}^2},$$

$$\sqrt{\Delta m_{21}^2} \ll \sqrt{\Delta m_{43}^2} \ll \sqrt{\Delta m_{41}^2}. \quad (58)$$

On the basis of (58), one can make the identification:

$$\Delta m_{21}^2 \equiv \Delta m_{\odot}^2, \quad \Delta m_{43}^2 \equiv \Delta m_{\text{atm}}^2, \quad \Delta m_{41}^2 \equiv \Delta m_{\text{SLB}}^2. \quad (59)$$

From (58) and (59) we get:

$$m_2 = \sqrt{m_1^2 + \Delta m_{\odot}^2}, \quad (60)$$

$$m_3 \simeq m_4 = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}, \quad (61)$$

where  $\Delta m_{\odot}^2$  takes values in the intervals given in Table 1 and the values  $\Delta m_{\text{SBL}}^2$  can have are given in eq. (7). In the expression for  $m_3 = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2 - \Delta m_{\text{atm}}^2}$  we have neglected terms  $\sim \Delta m_{\text{atm}}^2 / (m_1^2 + \Delta m_{\text{SBL}}^2) \ll 1$  and in this approximation  $|\langle m \rangle|$  does not depend on  $\Delta m_{\text{atm}}^2$ . The contribution of the indicated terms in  $|\langle m \rangle|$ , however, does not exceed  $10^{-4}$  eV. This follows from eqs. (7) and (9)-(10) and the fact that  $|U_{e3}|^2 \leq 1.0 \times 10^{-2}$  (see further). One will have  $m_1 \ll \sqrt{\Delta m_{\text{SBL}}^2}$  if

$$m_1 < 0.25 \sqrt{\Delta m_{\text{SBL}}^2} \Rightarrow m_1 < 0.1 \text{ eV}. \quad (62)$$

We find for the elements of interest of the lepton mixing matrix,  $U_{ej}$ :

i)  $|U_{e1}|$  and  $|U_{e2}|$  are related to the solar neutrino mixing angle  $\theta_{\odot}$ :

$$|U_{e1}|^2 = \cos^2 \theta_{\odot} \left(1 - \sum_{i=3}^4 |U_{ei}|^2\right), \quad (63)$$

$$|U_{e2}|^2 = \sin^2 \theta_{\odot} \left(1 - \sum_{i=3}^4 |U_{ei}|^2\right), \quad (64)$$

where  $\tan^2 \theta_{\odot}$  takes values in the regions reported in Table 1;

ii)  $|U_{e3}|$  and  $|U_{e4}|$  are constrained by the neutrino oscillation data from the short baseline (SBL) experiments LSND and BUGEY [35, 51]:

$$2.0 \times 10^{-4} \leq |U_{e3}|^2 + |U_{e4}|^2 \leq 1.0 \times 10^{-2}. \quad (65)$$

Using eq. (18) and neglecting  $|U_{e3}|^2 + |U_{e4}|^2$  with respect to 1 in eqs. (63) - (64), we obtain for the effective Majorana mass  $|\langle m \rangle|$ :

$$\begin{aligned} |\langle m \rangle| \simeq & |m_1 \cos^2 \theta_{\odot} + \sqrt{m_1^2 + \Delta m_{\odot}^2} \sin^2 \theta_{\odot} e^{i\alpha_{21}} \\ & + \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} (|U_{e3}|^2 e^{i\alpha_{31}} + |U_{e4}|^2 e^{i\alpha_{41}})| \end{aligned} \quad (66)$$

where  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$  are three CP-violating phases.

We will study next the magnitude of the contributions given by each of the four terms present in the r.h.s. of equation (66). Using the allowed values of  $\Delta m_{\text{SBL}}^2$  found in ref. [35] at 95% C.L. as well as the values  $\tan^2 \theta_{\odot}$  and  $\Delta m_{\odot}^2$  obtained in ref. [42] at 90% (99%) C.L. for  $\cos^2 \beta = 0.3$  [ $\cos^2 \beta = 0.5$ ] and taking into account the constraint (65), one finds if  $m_1 < 0.1$  eV:



$$0 \leq |< m >|_1 \equiv m_1 \cos^2 \theta_\odot \leq \begin{cases} 7.5 \text{ (8.0)}[6.7 \text{ (7.4)}] \times 10^{-2} \text{ eV} , & \text{LMA MSW;} \\ 0.1 \text{ (0.1)}[0.1 \text{ (0.1)}] \text{ eV} , & \text{SMA MSW;} \\ 6.0 \text{ (6.9)}[ \text{ (6.5)}] \times 10^{-2} \text{ eV} , & \text{LOW-QVO;} \end{cases} \quad (67)$$

$$|< m >|_2 \equiv \sqrt{m_1^2 + \Delta m_\odot^2} \sin^2 \theta_\odot \leq \begin{cases} 3.9 \text{ (5.5)}[3.3 \text{ (3.3)}] \times 10^{-2} \text{ eV} , & \text{LMA MSW;} \\ 1.0 \text{ (1.5)}[1.0 \text{ (1.5)}] \times 10^{-4} \text{ eV} , & \text{SMA MSW;} \\ 4.0 \text{ (7.4)}[ \text{ (6.9)}] \times 10^{-2} \text{ eV} , & \text{LOW-QVO;} \end{cases} \quad (68)$$

$$|< m >|_2 \equiv \sqrt{m_1^2 + \Delta m_\odot^2} \sin^2 \theta_\odot \geq \begin{cases} 10 \text{ (7.0)}[1.3 \text{ (1.2)}] \times 10^{-4} \text{ eV} , & \text{LMA MSW;} \\ 8.0 \text{ (4.0)}[8.0 \text{ (4.0)}] \times 10^{-7} \text{ eV} , & \text{SMA MSW;} \\ 13 \text{ (0.8)}[ \text{ (0.9)}] \times 10^{-5} \text{ eV} , & \text{LOW-QVO;} \end{cases} \quad (69)$$

$$0 \leq |< m >|_3 \equiv \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e3}|^2 \leq 5.0 \times 10^{-3} \text{ eV} ; \quad (70)$$

$$0 \leq |< m >|_4 \equiv \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 \leq 5.0 \times 10^{-3} \text{ eV} . \quad (71)$$

The upper bounds in eqs. (67), (68) and (70 (or (71))) will increase approximately by factors of 10, 3 and 2.2, respectively, if  $m_1 \simeq 1.0 \text{ eV}$ , while the lower bounds in eq. (69) will be larger by a factor of  $\sim 10$ .

This analysis shows that all contributions in  $|< m >|$ , eq. (66), can be of the same order. Thus, in the case under study all three Majorana CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$ , are relevant and must be taken into account in order to establish the allowed range for  $|< m >|$ , and, in particular, to answer the question regarding the possibility of cancellations between the different contributions.

Let us note that  $|< m >|_3$  and  $|< m >|_4$  are not independent since  $2.0 \times 10^{-4} \leq |U_{e3}|^2 + |U_{e4}|^2 \leq 1.0 \times 10^{-2}$ . As a consequence, at least one of the two contributions is of the order of  $10^{-3} \text{ eV}$  and they cannot both be smaller than  $10^{-4} \text{ eV}$ .

The terms  $|< m >|_1$  and  $|< m >|_2$  depend strongly on the values of  $m_1$  and  $\theta_\odot$ . More specifically we have:

1. for  $m_1 \leq 10^{-5} \text{ eV}$  and the LMA solution of the solar neutrino problem,  $|< m >|_1$  is negligible with respect to  $|< m >|_2$ . Indeed, one has  $|< m >|_1 \leq 0.05 |< m >|_2$ , while  $|< m >|_2$  can be of the same order as  $|< m >|_3$  and  $|< m >|_4$ , i.e.,  $|< m >|_2 \leq 10^{-2} \text{ eV}$ . The expression for  $|< m >|$  reduces to the sum of three terms with two relative phases  $(\alpha_{41} - \alpha_{21})$  and  $(\alpha_{31} - \alpha_{21})$ :

$$|< m >| \simeq |\sqrt{\Delta m_\odot^2} \sin^2 \theta_\odot + \sqrt{\Delta m_{\text{SBL}}^2} (|U_{e3}|^2 e^{i\alpha_{31}} + |U_{e4}|^2 e^{i\alpha_{41}}) e^{-i\alpha_{21}}| \quad (72)$$

For  $\max(\Delta m_\odot^2) = 2.0 \text{ (7.0)} \times 10^{-4} \text{ eV}^2$ , we have  $\sqrt{\Delta m_\odot^2} \sin^2 \theta_\odot \lesssim 0.7 \text{ (1.3)} \times 10^{-2} \text{ eV}$ . Since, as can be shown, the contribution of the term  $\sim \sqrt{\Delta m_{\text{SBL}}^2}$  does not exceed  $5.0 \times 10^{-3} \text{ eV}$ , for the maximal value of  $|< m >|$  one finds:  $|< m >| \lesssim 1.2 \text{ (1.8)} \times 10^{-2} \text{ eV}$ .

In the cases of the LOW-QVO and SMA solutions, both  $|< m >|_1$  and  $|< m >|_2$  are negligible with respect to  $|< m >|_3$  and  $|< m >|_4$  since  $|< m >|_1, |< m >|_2 \leq 10^{-4} \text{ eV}$ . In these cases only one relative CP-violating Majorana phase,  $(\alpha_{41} - \alpha_{31})$ , has effectively to be taken into account:

$$|< m >| \simeq |\sqrt{\Delta m_{\text{SBL}}^2} (|U_{e3}|^2 e^{i\alpha_{31}} + |U_{e4}|^2 e^{i\alpha_{41}})|. \quad (73)$$

However, one has  $|\langle m \rangle| \lesssim 5.0 \times 10^{-3} \text{ eV}$ .

2. For  $10^{-5} \text{ eV} \ll m_1 \ll 10^{-1} \text{ eV}$  all terms in eq. (66) can be of the same order and can give nonnegligible contributions to the effective Majorana mass  $|\langle m \rangle|$ . The three CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$ , have to be taken into account. Only in the case of the SMA solution,  $|\langle m \rangle|_2$  is negligible with respect to the other three terms.
3. If  $m_1 \gtrsim 10^{-1} \text{ eV}$  and for the LMA and LOW-QVO solutions,  $|\langle m \rangle|_1$  and  $|\langle m \rangle|_2$  give the dominant contributions in  $|\langle m \rangle|$ , the contribution of  $|\langle m \rangle|_3$  and  $|\langle m \rangle|_4$  being at least by an order of magnitude smaller. Thus, up to corrections  $\sim 5.0 \times 10^{-3} \text{ eV}$ , only the CP-violating phase  $\alpha_{21}$  is effectively relevant for the determination of  $|\langle m \rangle|$ , when  $|\langle m \rangle| \gtrsim 2 \times 10^{-2} \text{ eV}$ :

$$|\langle m \rangle| \gtrsim 2 \times 10^{-2} \text{ eV} : \quad |\langle m \rangle| \simeq m_1 |\cos^2 \theta_\odot + \sin^2 \theta_\odot e^{i\alpha_{21}}|. \quad (74)$$

Correspondingly, one finds:

$$|\langle m \rangle| \gtrsim 2 \times 10^{-2} \text{ eV} : \quad m_1 |\cos 2\theta_\odot| \leq |\langle m \rangle| \leq m_1. \quad (75)$$

In the case of the SMA solution,  $|\langle m \rangle|_2$ ,  $|\langle m \rangle|_3$  and  $|\langle m \rangle|_4$  are negligible in comparison with  $|\langle m \rangle|_1$  and we have:

$$|\langle m \rangle| \cong m_1 \cos^2 \theta_\odot \cong m_1. \quad (76)$$

In the future, with a better determination of the values of the relevant parameters, the allowed ranges for the different contributions will be considerably restricted and the analysis of the various possibilities will be simplified.

In the case of CP-invariance the massive Majorana neutrinos can have different CP-parities and it is possible to have cancellations between the different terms in eq. (66). Cancellations can occur also if the CP-invariance does not hold. Thus, in the case of the 2+2B neutrino mass spectrum under discussion one cannot obtain at present non-trivial lower bounds on  $|\langle m \rangle|$ . Majorana CP-violating phases can be present in  $|\langle m \rangle|$ , but this might not be recognizable from the measurement of  $|\langle m \rangle| \neq 0$  unless there exists a hierarchy between the different contributions in  $|\langle m \rangle|$ .

Furthermore, in the case of the 2+2B neutrino mass spectrum all the contributions tend to be relatively small. Therefore the maximal possible values of  $|\langle m \rangle|$  are of particular interest in view of the planned sensitivity of the the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments. These maximal values depend, obviously, on the solution of the solar neutrino problem. They can be found by assuming that CP-invariance holds and by taking the CP parities of all four massive Majorana neutrinos to be equal, and are reported in Fig. 6 as a function of  $m_1$ . For  $m_1$  negligible (e.g., a very light sterile neutrino) and  $\cos^2 \beta = 0.3$  [ $\cos^2 \beta = 0.5$ ], we get:

$$|\langle m \rangle| \leq \begin{cases} 8.9 \text{ (10) } [8.8 \text{ (8.8)}] \times 10^{-3} \text{ eV} , & \text{LMA MSW solution at 90\% (99\%) C.L.;} \\ 5.0 [5.0] \times 10^{-3} \text{ eV} & \text{SMA MSW solution;} \\ 5.0 [5.0] \times 10^{-3} \text{ eV} & \text{LOW-QVO solution.} \end{cases} \quad (77)$$

As  $m_1$  increases, the maximal values of  $|\langle m \rangle|$  also increase, reaching for  $m_1 \simeq 0.1 \text{ eV}$

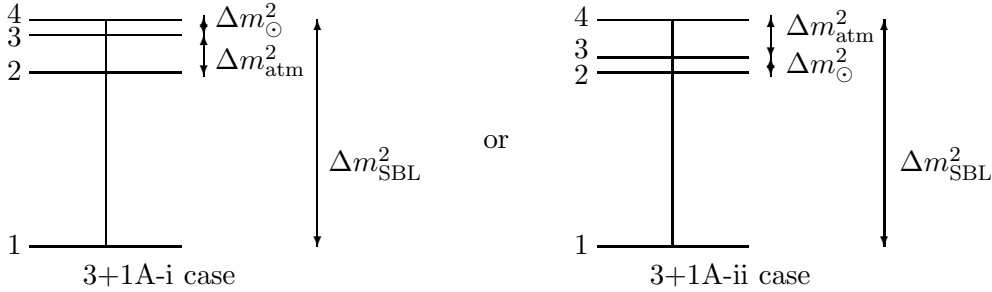
$$|\langle m \rangle| \leq 0.10 \text{ eV} \quad (78)$$

for all solutions of the solar neutrino problem. For  $m_1 \gtrsim 0.1 \text{ eV}$  we have  $\max(|\langle m \rangle|) \simeq m_1$ . One can have  $|\langle m \rangle| \gtrsim 3.0 \times 10^{-2} \text{ eV}$  for the 2+2B type of neutrino mass spectrum only if  $m_1 \gtrsim 10^{-2}$

eV. Values of  $|\langle m \rangle| \gtrsim 10^{-2}$  eV are in the region of the planned sensitivity of the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments. As it follows from Fig. 6, a measured value of  $|\langle m \rangle| \gtrsim 0.03$  eV in the case of the neutrino mass spectrum under discussion would imply a lower limit on the mass  $m_1$ ,  $|\langle m \rangle| \gtrsim 10^{-2}$  eV; and it would essentially determine the value of  $m_1$  if the SMA MSW solution of the solar neutrino problem turns out to be the correct one.

## 6 The 3+1A mass spectrum

The 3+1A mass spectrum is characterized by three nearly degenerate neutrinos,  $\nu_{2,3,4}$ , having masses sufficiently larger than the fourth one,  $\nu_1$ :  $m_1 < (\ll) m_2, m_3, m_4$ . There exist two possibilities which can be presented schematically as follows:



One has  $m_1 \ll m_4$ , if  $m_1 < 0.1$  eV. These patterns of neutrino masses can also be characterized by the inequalities:

$$\Delta m_{43}^2 \ll \Delta m_{32}^2 \ll \Delta m_{41}^2, \quad \text{3+1A-i case;} \quad (79)$$

$$\Delta m_{32}^2 \ll \Delta m_{43}^2 \ll \Delta m_{41}^2, \quad \text{3+1A-ii case.} \quad (80)$$

Using relation (79) in the 3+1A-i case, one can make the identification:

$$\Delta m_{43}^2 \equiv \Delta m_{\odot}^2, \quad \Delta m_{32}^2 \equiv \Delta m_{\text{atm}}^2, \quad \Delta m_{41}^2 \equiv \Delta m_{\text{SBL}}^2. \quad (81)$$

The corresponding constraints on the elements  $|U_{ej}|$  of the neutrino mixing matrix  $U$  read:

- i)  $|U_{e1}|$  is limited by the data of the SBL experiments [35, 51]:  $2.0 \times 10^{-4} \leq |U_{e1}|^2 < 1.0 \times 10^{-2}$ ;
- ii)  $|U_{e2}|$  should satisfy the CHOOZ limit [55, 58]:  $|U_{e2}|^2 < 0.08$  (99% C.L.);
- iii)  $|U_{e3}|$  and  $|U_{e4}|$  are related to the solar neutrino mixing angle  $\theta_{\odot}$ :

$$|U_{e3}|^2 = \cos^2 \theta_{\odot} \left( 1 - \sum_{i=1}^2 |U_{ei}|^2 \right), \quad (82)$$

$$|U_{e4}|^2 = \sin^2 \theta_{\odot} \left( 1 - \sum_{i=1}^2 |U_{ei}|^2 \right). \quad (83)$$

Using eq. (81) one finds :

$$m_2 = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2 - \Delta m_{\odot}^2 - \Delta m_{\text{atm}}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2 - \Delta m_{\odot}^2}, \quad m_4 = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}. \quad (84)$$

Neglecting terms  $\sim \Delta m_{\odot}^2 / \Delta m_{\text{SBL}}^2$  and  $\sim \Delta m_{\text{atm}}^2 / \Delta m_{\text{SBL}}^2$  in  $m_{2,3}$ , whose contributions in  $|\langle m \rangle|$  do not exceed respectively  $10^{-3}$  eV and  $1.5 \times 10^{-3}$  eV, we get:

$$m_2 \simeq m_3 \simeq m_4 = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} . \quad (85)$$

The 3+1A-ii neutrino mass spectrum can be obtained from the 3+1A-i one by requiring that  $\Delta m_{43}^2 \equiv \Delta m_{\text{atm}}^2$ ,  $\Delta m_{32}^2 \equiv \Delta m_{\odot}^2$  and by interchanging  $|U_{e2}|^2$  and  $|U_{e4}|^2$ . However, in the approximation (85) we are working (i.e., neglecting terms smaller than  $\sim 10^{-3}$  eV in  $|\langle m \rangle|$ ),  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\odot}^2$  do not enter into the expression for  $|\langle m \rangle|$ . Since in this approximation  $m_2 \simeq m_3 \simeq m_4$ , we have also the freedom to interchange  $|U_{e2}|^2$  and  $|U_{e4}|^2$ , obtaining the expression for  $|\langle m \rangle|$  in the case of the 3+1A-i spectrum. Consequently, as far as the approximation (85) holds, i.e., up to corrections  $\sim 10^{-3}$  eV, the 3+1A-i and the 3+1A-ii patterns of neutrino masses lead to the same predictions for the effective Majorana mass  $|\langle m \rangle|$ . In what follows we will consider the 3+1A-i spectrum only and will refer generically to the two spectra as the 3+1A spectrum.

In the 3+1A schemes under discussion, as like in the 2+2A scheme, the neutrino mass  $m_{\nu_e}$  measured in the  $^3\text{H}$   $\beta$ -decay experiments is given, up to correction which do not exceed  $\sim 10^{-2}$  eV, by:

$$m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} . \quad (86)$$

From the results of the analysis of the LSND data [35] it follows that  $m_{\nu_e} \gtrsim 0.4$  eV for any value of  $m_1$ <sup>11</sup>. Thus, the possibility that the neutrino mass spectrum is of the 3+1A type can also be tested in the KATRIN  $^3\text{H}$   $\beta$ -decay experiment [67]. The observation made for the case of the 2+2A spectrum, that a measurement of  $m_{\nu_e} \gtrsim 0.4$  eV and a more precise knowledge of  $\Delta m_{\text{SBL}}^2$  would permit to determine the value of  $m_1$  and would allow to fix the values of  $m_{2,3,4}$  as well, is valid also for the 3+1A spectrum.

For  $m_1 < 0.1$  eV we have  $m_1|U_{e1}|^2 < 10^{-3}$  eV and in the approximation we have adopted this term in  $|\langle m \rangle|$  can be neglected. If  $m_1 \simeq 1.0$  eV, the contribution of the neglected term in  $|\langle m \rangle|$  would not exceed  $10^{-2}$  eV. Since  $|U_{e1}|^2 \leq 10^{-2}$ , one can neglect also  $|U_{e1}|^2$  in comparison with  $(1 - |U_{e2}|^2) > 0.91$ . Correspondingly, the expression for  $|\langle m \rangle|$  reduces to:

$$|\langle m \rangle| \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} | |U_{e2}|^2 + (1 - |U_{e2}|^2)(\cos^2 \theta_{\odot} e^{i(\alpha_3 - \alpha_2)} + \sin^2 \theta_{\odot} e^{i(\alpha_4 - \alpha_2)}) |. \quad (87)$$

Thus, in the case of 3+1A neutrino mass spectrum under study and up to terms which are not larger than  $\sim 10^{-3}$  eV and/or  $m_1|U_{e1}|^2$ ,  $|\langle m \rangle|$  depends on  $\sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$ ,  $\theta_{\odot}$  and  $|U_{e2}|^2$  and on two Majorana CP-violating phases (or phase differences)  $(\alpha_3 - \alpha_2) = (\alpha_{31} - \alpha_{21}) \equiv \alpha_{32}$  and  $(\alpha_4 - \alpha_2) = (\alpha_{41} - \alpha_{21}) \equiv \alpha_{42}$ .

It follows from eq. (87) that the expression for  $|\langle m \rangle|$  for the mass spectrum under discussion is very similar to the expression for  $|\langle m \rangle|$  in the case of the three quasi-degenerate neutrinos, which has been analyzed recently in detail in ref. [19] (see Section 6 in [19]). More specifically,

- i) the common neutrino mass  $m$ , which appears as an overall mass scale in  $|\langle m \rangle|$  in the case of three quasi-degenerate Majorana neutrinos (see, e.g., [19]), corresponds to  $\sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$  in eq. (87). Thus, we can make the identification  $m \equiv \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$ , where  $m$  in the 3-neutrino mixing analysis can vary in the interval [19]  $0.1 \text{ eV} \leq m \leq 2.5 \text{ eV}$  ;
- ii) in both cases the expression for  $|\langle m \rangle|$  contains two Majorana CP-violating phases:  $\alpha_{32} \equiv (\alpha_{31} - \alpha_{21})$  and  $\alpha_{42} \equiv (\alpha_{41} - \alpha_{21})$  are present in the 4-neutrino mixing expression (87),

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<sup>11</sup>That  $m_{\nu_e}$  is not necessarily small in certain 3+1 schemes was noticed also in [80].

while  $\alpha_{31}$  and  $\alpha_{21}$  are present in the corresponding 3-neutrino mixing one. As was shown in ref. [19], it would be possible, in principle, to constrain the CP-violating phases  $\alpha_{31}$  and  $\alpha_{21}$  if the values of the relevant parameters entering into the  $3-\nu$  expression for  $|\langle m \rangle|$ ,  $m$ ,  $|U_{e1}|$  and  $\theta_\odot$ , as well as the value of  $|\langle m \rangle|$ , were known with sufficient accuracy (see eq. (82) and Fig. 16 in [19]). The Majorana phases  $\alpha_{31}$  and  $\alpha_{21}$  are the only possible source of CP-violation effects in  $|\langle m \rangle|$  in the case of 3-neutrino mixing. If 4-neutrino mixing takes place and the neutrino mass spectrum is of the 3+1A type, it would still be possible, in principle, to constrain two of the physical Majorana CP-violating phases, but not the third one,  $\alpha_{21}$ , which can be an additional source of CP-violation.

Further, we have  $|U_{e2}|^2 < 0.08$  according to the (99% C.L.) results of ref. [55]. Therefore, the terms containing  $|U_{e2}|^2$  as a factor in eq. (87) give non-dominant contribution to  $|\langle m \rangle|$  and the results for  $|\langle m \rangle|$  in the case of the 3+1A spectrum are quite similar to those derived for the 2+2A spectrum. Actually, the expressions for  $|\langle m \rangle|$  in the two cases coincide in the limit of  $|U_{e2}|^2 = 0$ . For  $|U_{e2}|^2 = 0$  the difference in the predictions for  $|\langle m \rangle|$  arises only due to the fact that, for the 2+2A neutrino mass spectrum one has to use the results of a 4-neutrino mixing analysis of the relevant data, while if the spectrum is of the 3+1A type, the results of the 3-neutrino mixing analysis of the atmospheric, solar and CHOOZ neutrino oscillation data should be utilized, as is discussed in Section 2.

We will treat the 3+1A-i case in what follows taking into account the  $|U_{e2}|^2 \neq 0$  terms in  $|\langle m \rangle|$ , i.e., using expression (87) for  $|\langle m \rangle|$ . First we will consider the CP-conserving cases.

**Case A.** If the neutrinos  $\nu_3$  and  $\nu_4$  have the same CP-parities,  $\phi_3 = \phi_4 = \pm\phi_2$  (i.e.  $\alpha_{41} = \alpha_{31} = 0, \pm\pi$ ,  $\alpha_{21} = 0, \pm\pi$ ),  $|\langle m \rangle|$  is given by:

$$|\langle m \rangle| = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} (1 - |U_{e2}|^2 \pm |U_{e2}|^2). \quad (88)$$

Using the data on  $\Delta m_{\text{SBL}}^2$  and  $|U_{e2}|^2$  obtained respectively in ref. [35] (at 95% C.L.) and in ref. [58] (at 90% C.L.), we find that for any  $m_1$ :

$$|\langle m \rangle| \geq 0.4 \text{ eV}, \quad \text{for } \phi_3 = \phi_4 = \phi_2; \quad (89)$$

$$|\langle m \rangle| \geq 0.3 \text{ eV}, \quad \text{for } \phi_3 = \phi_4 = -\phi_2. \quad (90)$$

If  $m_1^2 \ll \Delta m_{\text{SBL}}^2$ , we get for the maximal allowed values of  $|\langle m \rangle|$ :  $|\langle m \rangle| \leq 1.4 \text{ eV}$  for  $\phi_3 = \phi_4 = \phi_2$ , and  $|\langle m \rangle| \leq 1.1 \text{ eV}$  if  $\phi_3 = \phi_4 = -\phi_2$ . As in the similar case of 2+2A neutrino mass spectrum, the experimental upper limits [29, 30], eqs. (4) and (5), exclude part (if not all) of the allowed regions of values (89) and (90) of  $|\langle m \rangle|$ . An improvement of these limits by a factor  $\sim (3-4)$  would essentially rule out the considered possibility.

**Case B.** If the CP-parities of the neutrinos  $\nu_{2,3,4}$  satisfy  $\phi_3 = -\phi_4 = \pm\phi_2$  (i.e., if  $\alpha_{41} = \alpha_{31} + \pi = 0, \pm\pi$ ,  $\alpha_{21} = 0, \pm\pi$ ), one has:

$$|\langle m \rangle| = \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |(1 - |U_{e2}|^2) \cos 2\theta_\odot \pm |U_{e2}|^2|. \quad (91)$$

The predictions for the effective Majorana mass  $|\langle m \rangle|$  depend on the solution of the solar neutrino problem:

1. In the case of the LMA MSW solution, the 90% (99%) C.L. results obtained in ref. [55] do not exclude the possibility of cancellations between the different terms in eq. (87):

$$0 \text{ (0) eV} \leq |\langle m \rangle| \leq 9.8 \text{ (10)} \times 10^{-1} \text{ eV}, \quad \text{for } \phi_3 = -\phi_4 = \phi_2; \quad (92)$$

$$0 \text{ (0) eV} \leq |\langle m \rangle| \leq 9.5 \text{ (10)} \times 10^{-1} \text{ eV}, \quad \text{for } \phi_3 = -\phi_4 = -\phi_2. \quad (93)$$

The maximal values of  $|\langle m \rangle|$  correspond to  $m_1 < 0.1$  eV. They will be larger by a factor of  $\sim 1.2$  if  $m_1 \simeq 1.0$  eV. A better determination of the value of  $\theta_\odot$  will be crucial for reducing the allowed ranges of  $|\langle m \rangle|$  in this case. This might lead, in particular, to a non-trivial lower bound on  $|\langle m \rangle|$ .

2. For the SMA MSW solution one has  $\cos 2\theta_\odot \simeq 1$  and we obtain the following intervals of allowed values of  $|\langle m \rangle|$  corresponding to the 90% C.L. (99% C.L.) solution regions and  $m_1^2 \ll \Delta m_{\text{SBL}}^2$  :

$$0.4 \text{ eV} \leq |\langle m \rangle| \leq 1.4 \text{ eV} , \quad \text{for } \phi_3 = -\phi_4 = \phi_2; \quad (94)$$

$$0.3 \text{ eV} \leq |\langle m \rangle| \leq 1.1 \text{ eV} , \quad \text{for } \phi_3 = -\phi_4 = -\phi_2. \quad (95)$$

This case cannot be distinguished from the earlier considered case A,  $\phi_3 = \phi_4 = \pm\phi_2$ . The comments made at the end of the discussion of the case A are valid also in the present case.

3. The values of  $\theta_\odot$ , corresponding to the 90% C.L. ( 99% C.L.) LOW-QVO solution, allow cancellations between the different terms in  $|\langle m \rangle|$  to take place and for  $m_1^2 \ll \Delta m_{\text{SBL}}^2$  one finds:

$$0 \text{ (0) eV} \leq |\langle m \rangle| \leq 7.0 \text{ (9.7)} \times 10^{-1} \text{ eV} , \quad \phi_3 = -\phi_4 = \phi_2; \quad (96)$$

$$0 \text{ (0) eV} \leq |\langle m \rangle| \leq 6.7 \text{ (9.4)} \times 10^{-1} \text{ eV} , \quad \phi_3 = -\phi_4 = -\phi_2. \quad (97)$$

As in the cases considered above, the upper limits are by a factor of  $\sim 1.2$  larger for  $m_1 \simeq 1.0$  eV.

We see that in all cases of CP-conservation  $|\langle m \rangle|$  can take values in the region  $|\langle m \rangle| \gtrsim 0.01$  eV which can be tested in the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments.

We show in Fig. 7 the regions of allowed values of  $|\langle m \rangle|$  as a function of  $\sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} \cong \sqrt{\Delta m_{\text{SBL}}^2}$  for the LMA and LOW-QVO solutions of the solar neutrino problem. The regions were obtained assuming CP-invariance does not hold as well as for the different cases of CP-invariance considered above. The CP-violation region includes all the CP-conserving ones: all regions marked by different grey scales are allowed if CP-invariance does not hold. The region marked by dark-grey color is the “just-CP-violating” region: it can be spanned by the values of  $|\langle m \rangle|$  only if the CP-symmetry is violated. In a large region of the relevant parameter space one can have values of  $|\langle m \rangle| \geq 0.1$  eV, which can be tested by the current and future  $(\beta\beta)_{0\nu}$ -decay experiments. A positive result obtained in these experiments and a less ambiguous determination of  $\Delta m_{\text{SBL}}^2$  might allow to establish, provided the neutrino mass spectrum is of the 3+1A type, whether the CP-symmetry is violated, and to get information on the relative CP-parities of the massive Majorana neutrinos  $\nu_{2,3,4}$  if the phases  $\alpha_{32}$  and  $\alpha_{42}$  take their CP-conserving values. We would like to remind the reader that for the spectrum under discussion, the CP-violating phase  $\alpha_{21}$  cannot be constrained by the data on  $|\langle m \rangle|$ ,  $\Delta m_{\text{SBL}}^2$  and  $\tan^2 \theta_\odot$  and can be a source of CP-violation in other processes.

According to eq. (87),  $|\langle m \rangle|$  should exhibit a rather strong dependence on  $\cos 2\theta_\odot$ . This dependence is illustrated in Fig. 8, where we plot  $|\langle m \rangle| / \sqrt{\Delta m_{\text{SBL}}^2}$  as a function of  $\cos 2\theta_\odot$ . The “just-CP-violating” region is shown in Fig. 8 in dark-grey color.

Equation (87) allows to find a relation between the CP-violating phases  $(\alpha_3 - \alpha_2) \equiv \alpha_{32}$  and  $(\alpha_4 - \alpha_2) \equiv \alpha_{42}$  and the observable quantities  $|\langle m \rangle|$ ,  $|U_{e2}|^2$ ,  $m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$  and  $\theta_\odot$ :

$$1 - \frac{|\langle m \rangle|^2}{(m_1^2 + \Delta m_{\text{SBL}}^2)} = 2|U_{e2}|^2(1 - |U_{e2}|^2) (1 - \cos^2 \theta_\odot \cos \alpha_{32} - \sin^2 \theta_\odot \cos \alpha_{42}) \\ + (1 - |U_{e2}|^2)^2 \sin^2 2\theta_\odot \sin^2 (\alpha_{32} - \alpha_{42}) \quad (98)$$

A sufficiently accurate measurement of  $|\langle m \rangle|$  and of the other observables which enter into the above equation might allow to obtain significant constraints on the phases  $\alpha_{32}$  and  $\alpha_{42}$  and thus to get information on the CP-violation in the lepton sector. Figures 9 and 10 illustrate this possibility: we show in these figures the allowed values of  $\cos \alpha_{32}$  and  $\cos \alpha_{42}$  for a set of values of  $|U_{e2}|^2 = 0.0, 0.01, \text{ and } 0.04, 0.08$ , respectively. We have used the best fit value for  $\cos 2\theta_\odot$  [55] and for each value of  $|U_{e2}|^2$  we consider a set of values of  $|\langle m \rangle|$  given by  $|\langle m \rangle| = \sqrt{0.15 + 0.1n} \sqrt{\Delta m_{\text{SBL}}^2}$  with  $n = 1, 2, \dots, 8$ .

The constraint (98) will simplify considerably if the mixing parameter  $|U_{e2}|^2$ , which is limited by the CHOOZ data, turned out to be relatively small, say  $|U_{e2}|^2 \leq 10^{-2}$ . Let us note that the MINOS experiment [60] is planned to be sensitive to values of  $|U_{e2}|^2 \gtrsim 5 \times 10^{-3}$ . Neglecting terms  $\sim |U_{e2}|^2$  with respect to 1 in eq. (98) we get:

$$\sin^2(\alpha_{32} - \alpha_{42})2 \simeq 1 \sin^2 2\theta_\odot \left( 1 - \frac{|\langle m \rangle|^2}{(m_1^2 + \Delta m_{\text{SBL}}^2)} \right) \simeq 1 \sin^2 2\theta_\odot \left( 1 - \frac{|\langle m \rangle|^2}{m_{\nu_e}^2} \right). \quad (99)$$

The measurement of  $|\langle m \rangle| \neq 0$  and of  $m_{\nu_e} \gtrsim 0.4$  eV in this case can provide information on one of the Majorana CP-violating phases,  $(\alpha_{32} - \alpha_{42}) = \alpha_{34}$ .

Equation (98) takes a simpler form in the case of the SMA MSW solution of the solar neutrino problem as well. For the SMA MSW solution one has [55]  $\sin^2 \theta_\odot \lesssim 2.5 \times 10^{-3}$  eV<sup>2</sup> and neglecting terms  $\sim \sin^2 \theta_\odot$ , eq. (98) reduces to:

$$4|U_{e2}|^2(1 - |U_{e2}|^2) \sin^2 \alpha_{32}2 \simeq \left( 1 - \frac{|\langle m \rangle|^2}{(m_1^2 + \Delta m_{\text{SBL}}^2)} \right) \simeq \left( 1 - \frac{|\langle m \rangle|^2}{m_{\nu_e}^2} \right). \quad (100)$$

Note that, as in the previous case, only one Majorana CP-violating phase enters into eq. (100) and could be constrained by using data on  $|\langle m \rangle|$ ,  $m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$  and  $|U_{e2}|^2$ . For  $m_1$  not known, eq. (100) allows to obtain a correlated constraint on  $m_1$  and  $\alpha_{32}$ .

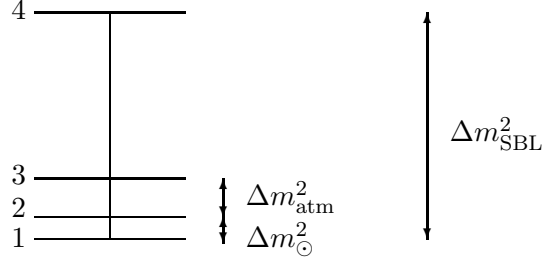
If the SMA MSW solution is the correct solution of the solar neutrino problem and  $|U_{e2}|^2$  is rather small, e.g.,  $|U_{e2}|^2 \lesssim 5 \times 10^{-3}$ , the measurement of  $|\langle m \rangle| \gtrsim 10^{-2}$  eV will hardly provide any information on the violation of the CP-symmetry in the lepton sector, since under the above circumstances we would have for the 3+1A neutrino mass spectrum under discussion:

$$|\langle m \rangle| \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} (1 + O(10^{-2})) \simeq m_{\nu_e}. \quad (101)$$

However, as eq. (101) indicates, such a measurement can give information on the mass of the lightest neutrino  $m_1$ . Let us remind the reader that similar result holds for the SMA MSW solution (or in the CP-conserving case A) if the neutrino mass spectrum is of the 2+2A type.

## 7 The 3+1B mass spectrum

The 3+1B neutrino mass spectrum corresponds to three nearly-degenerate neutrinos  $\nu_{1,2,3}$  with masses sufficiently smaller than the mass of the fourth one,  $m_1 \simeq m_2 \simeq m_3 < (\ll) m_4$ , as is indicated graphically below:



This neutrino mass spectrum can also be characterized by:

$$\Delta m_{21}^2 \ll \Delta m_{32}^2 \ll \Delta m_{41}^2 \quad (102)$$

Equation (102) allows one to relate the neutrino mass-squared differences  $\Delta m_{jk}^2$  to those determined from the solar, atmospheric and LSND neutrino oscillation data:

$$\Delta m_{21}^2 \equiv \Delta m_{\odot}^2, \quad \Delta m_{32}^2 \equiv \Delta m_{\text{atm}}^2, \quad \Delta m_{41}^2 \equiv \Delta m_{\text{SBL}}^2. \quad (103)$$

The three neutrinos  $\nu_{1,2,3}$  can have a hierarchical mass spectrum, a spectrum with partial hierarchy or a quasi-degenerate spectrum (see, e.g., [19]).

Using (103) we can express the masses  $m_{2,3,4}$  as functions of  $m_1$  and the measured neutrino mass-squared differences:

$$\begin{aligned} m_2 &= \sqrt{m_1^2 + \Delta m_{\odot}^2}, \\ m_3 &= \sqrt{m_1^2 + \Delta m_{\odot}^2 + \Delta m_{\text{atm}}^2}, \\ m_4 &= \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}. \end{aligned} \quad (104)$$

We will require also that, as in the case of the 3-neutrino mass spectrum with hierarchy or partial mass hierarchy,  $m_1$  satisfies [19]:  $m_1 \leq 0.2$  eV. For the spectrum with partial mass hierarchy one has [19]  $0.02$  eV  $\lesssim m_1 \leq 0.2$  eV, while for the hierarchical spectrum  $m_1 \ll 0.02$  eV. In the latter case the contribution of  $m_1$  to  $\langle m \rangle$  is negligible. Given the allowed values of  $\Delta m_{\text{SBL}}^2$ , eq. (7), and eq. (104), the inequality  $m_1 \leq 0.2$  eV implies  $m_1^2 \ll m_4^2$ . For  $m_1 > 0.2$  eV one gets essentially a quasi-degenerate  $\nu_{1,2,3}$  mass spectrum. We will discuss the predictions for  $\langle m \rangle$  in this latter case at the end of the Section.

For the elements of interest of the neutrino mixing matrix,  $|U_{ej}|$ , one finds:

- i)  $|U_{e1}|$  and  $|U_{e2}|$  are related to the solar mixing angle  $\theta_{\odot}$ :

$$\begin{aligned} |U_{e1}|^2 &= \cos^2 \theta_{\odot} (1 - |U_{e3}|^2), \\ |U_{e2}|^2 &= \sin^2 \theta_{\odot} (1 - |U_{e3}|^2); \end{aligned} \quad (105)$$

- ii)  $|U_{e3}|^2$  should satisfy the CHOOZ upper limit which at 99% C.L. reads [58, 55]:  $|U_{e3}|^2 < 0.08$ ;  
iii)  $|U_{e4}|^2$  is constrained by the data from the SBL experiments [35, 51] and we have  $2 \times 10^{-4} < |U_{e4}|^2 < 1 \times 10^{-2}$ .

Using eqs. (18) and (105), one can express the effective Majorana mass  $\langle m \rangle$  in terms of the quantities measured in the neutrino oscillation experiments,  $m_1$  and the Majorana CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$ :

$$\begin{aligned} \langle m \rangle &\simeq |m_1(1 - |U_{e3}|^2) \cos^2 \theta_{\odot} + \sqrt{m_1^2 + \Delta m_{\odot}^2} (1 - |U_{e3}|^2) \sin^2 \theta_{\odot} e^{i\alpha_{21}} \\ &\quad + \sqrt{m_1^2 + \Delta m_{\odot}^2 + \Delta m_{\text{atm}}^2} |U_{e3}|^2 e^{i\alpha_{31}} + \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 e^{i\alpha_{41}}|. \end{aligned} \quad (106)$$



Since  $|\langle m \rangle|$  depends on three CP-violating phases in this case, the general analysis of the allowed values of  $|\langle m \rangle|$  is rather complicated. It can be simplified if we consider first the absolute value of the sum of the first three contributions in eq. (106),  $|\langle m \rangle|_{3-\nu}$ :

$$|\langle m \rangle|_{3-\nu} \equiv |m_1(1 - |U_{e3}|^2) \cos^2 \theta_\odot + \sqrt{m_1^2 + \Delta m_\odot^2} (1 - |U_{e3}|^2) \sin^2 \theta_\odot e^{i\alpha_{21}} + \sqrt{m_1^2 + \Delta m_\odot^2 + \Delta m_{\text{atm}}^2} |U_{e3}|^2 e^{i\alpha_{31}}|. \quad (107)$$

The term  $|\langle m \rangle|_{3-\nu}$  of interest coincides with the effective Majorana mass for the neutrino mass spectrum with hierarchy or with partial mass hierarchy in the 3-neutrino mixing case, which have been studied in detail recently in ref. [19] (see Sections 4 and 7 in [19]). It was found in [19] that  $|\langle m \rangle|_{3-\nu}$  is constrained to lie in the intervals

$$\begin{aligned} 0 \leq |\langle m \rangle|_{3-\nu} &\leq 7.4 (14) \times 10^{-3} \text{ eV}, & \text{LMA MSW solution;} \\ 0 \leq |\langle m \rangle|_{3-\nu} &\leq 1.9 (3.6) \times 10^{-3} \text{ eV}, & \text{SMA MSW solution;} \\ 0 \leq |\langle m \rangle|_{3-\nu} &\leq 3.5 (3.6) \times 10^{-3} \text{ eV}, & \text{LOW-QVO solution,} \end{aligned} \quad (108)$$

for negligible  $m_1$  (hierarchical  $3 - \nu$  spectrum), and

$$\begin{aligned} 8.0 \times 10^{-4} (0) &\leq |\langle m \rangle|_{3-\nu} \leq 0.2 (0.2) \text{ eV}, & \text{LMA MSW solution;} \\ 1.6 (1.5) \times 10^{-2} &\leq |\langle m \rangle|_{3-\nu} \leq 0.2 (0.2) \text{ eV}, & \text{SMA MSW solution;} \\ 0 (0) &\leq |\langle m \rangle|_{3-\nu} \leq 0.2 (0.2) \text{ eV}, & \text{LOW-QVO solution,} \end{aligned} \quad (109)$$

for the spectrum with partial mass hierarchy, i.e., for  $0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV}$ . The values (the values in brackets) in eqs. (108) and (109) correspond to 90% (99%) C.L. solution regions [19]. The upper bounds in eq. (109) correspond to the maximal assumed value of  $m_1$ :  $\max(|\langle m \rangle|_{3-\nu}) = \max(m_1)$ . The fourth term in eq. (106),  $|\langle m \rangle|_4$ , is constrained by the SBL data [35] (at 95% C.L.):

$$3.0 \times 10^{-4} \text{ eV} \leq |\langle m \rangle|_4 \equiv \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 \leq 5.0 \times 10^{-3} \text{ eV}. \quad (110)$$

It follows from (108) and (110) that for the 3+1B spectrum one can have  $|\langle m \rangle| \gtrsim 3 \times 10^{-2} \text{ eV}$  only if  $m_1 \gtrsim 2 \times 10^{-2} \text{ eV}$  and that a measured value of  $|\langle m \rangle| \gtrsim 4 \times 10^{-2} \text{ eV}$  would imply:

$$|\langle m \rangle| \gtrsim 4 \times 10^{-2} \text{ eV} : \quad |\langle m \rangle| \simeq m_1(1 - |U_{e3}|^2) (1 - \sin^2 2\theta_\odot \sin^2 \alpha_{21}/2). \quad (111)$$

For  $\sin^2 2\theta_\odot \sin^2(\alpha_{21}/2)$  sufficiently smaller than 1 (e.g.,  $\sin^2 2\theta_\odot \sin^2(\alpha_{21}/2) \lesssim 0.5$ ) the above relation offers the possibility to determine the mass of the lightest neutrino  $m_1$ . The indicated condition is realized, e.g., for the SMA MSW solution of the solar neutrino problem ( $\sin^2 2\theta_\odot \ll 1$ ) and/or for  $\alpha_{21} = 2\pi k$ ,  $k = 0, 1, \dots$

The effective Majorana mass  $|\langle m \rangle|$  can be re-written in terms of  $|\langle m \rangle|_{3-\nu}$  and  $|\langle m \rangle|_4$  as follows:

$$|\langle m \rangle| = \left| |\langle m \rangle|_{3-\nu} e^{i(\gamma - \alpha_1)} + |\langle m \rangle|_4 e^{i\alpha_{41}} \right| \quad (112)$$

where  $\gamma$  is the overall CP-violating phase of the sum of the first three terms. Let us note that  $\gamma$  cannot be simply related to the CP-violating phases  $\alpha_{21}$  and  $\alpha_{31}$  entering into the expression for the sum of the first three terms in eq. (106). The physical meaning of the phase  $\gamma$  becomes more clear from Fig. 1, where we have represented graphically the term  $|\langle m \rangle|_{3-\nu}$  in the complex plane. As is shown in Fig. 1, the phase  $(\gamma - \alpha_1)$  is determined not only by  $\alpha_{21}$  and  $\alpha_{31}$ , but depends also on  $|\langle m \rangle|_1$ ,  $|\langle m \rangle|_2$  and  $|\langle m \rangle|_3$  - the absolute values of the three terms whose sum is

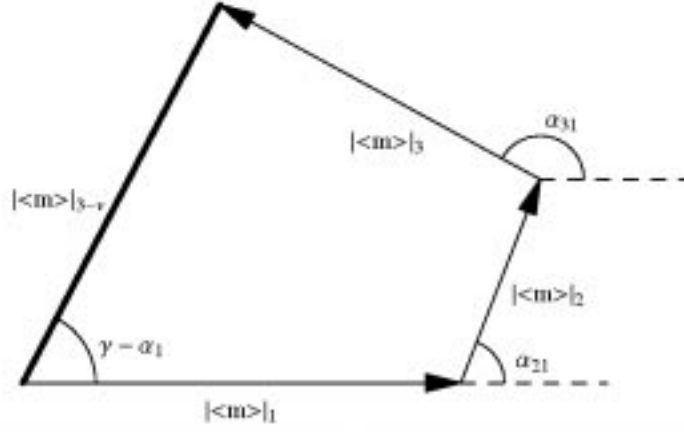


Figure 1: The contribution  $|\langle m \rangle|_{3-\nu}$  to the effective Majorana mass represented as the absolute value of the vector sum of the three contributions  $|\langle m \rangle|_1$ ,  $|\langle m \rangle|_2$  and  $|\langle m \rangle|_3$  in eqs. (107), each of which is expressed as a vector in the complex plane (see text for details). The CP-violating phases  $\alpha_{21}$  and  $\alpha_{31}$  shown in the figure can vary between 0 and  $2\pi$ . The effective CP-violating phase  $(\gamma - \alpha_1)$  is also shown in the figure.

$\langle m \rangle_{3-\nu}$ . If CP is conserved,  $\gamma$  and  $\alpha_4$  can be either 0 or  $\pm\pi$ . The phase  $(\gamma - \alpha_1)$  determines in the case of CP-conservation the sign of the contribution of the term  $\langle m \rangle_{3-\nu}$ , while  $(\gamma - \alpha_1 - \alpha_{41})$  fixes the relative sign of  $\langle m \rangle_{3-\nu}$  and  $\langle m \rangle_4$ . According to the different CP-conserving values the phases  $(\gamma - \alpha_1)$  and  $\alpha_{41}$  can assume, we have two possibilities.

**Case A.** If  $\gamma - \alpha_1 = \alpha_{41} = 0, \pm\pi$ ,  $|\langle m \rangle|$  is bounded to lie in the intervals

$$\begin{aligned} 3.0 \text{ (3.0)} \times 10^{-4} \text{ eV} &\leq |\langle m \rangle| \leq 1.2 \text{ (1.9)} \times 10^{-2} \text{ eV}, & \text{LMA MSW solution;} \\ 3.0 \text{ (3.0)} \times 10^{-4} \text{ eV} &\leq |\langle m \rangle| \leq 6.9 \text{ (8.6)} \times 10^{-3} \text{ eV}, & \text{SMA MSW solution;} \\ 3.0 \text{ (3.0)} \times 10^{-4} \text{ eV} &\leq |\langle m \rangle| \leq 8.5 \text{ (8.6)} \times 10^{-3} \text{ eV}, & \text{LOW-QVO solution,} \end{aligned} \quad (113)$$

if  $m_1$  is negligible (hierarchical  $3-\nu$  mass spectrum). For  $0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV}$  ( $3-\nu$  mass spectrum with partial hierarchy [19]) one gets

$$3.0 \text{ (3.0)} \times 10^{-4} \text{ eV} \leq |\langle m \rangle| \leq 0.2 \text{ (0.2)} \text{ eV} \quad (114)$$

for all solutions of the solar neutrino problem, where  $\max(|\langle m \rangle|) \simeq m_1$ . Equation (114) was obtained by using the 90% C.L. (95% C.L.) allowed values of  $\theta_\odot$ ,  $\Delta m_\odot^2$  and  $|U_{e3}|^2$  from ref. [55] and the allowed values of  $\Delta m_{\text{SBL}}^2$  and  $|U_{e4}|^2$  at 95% C.L. from ref. [35].

**Case B.** If  $(\gamma - \alpha_1)$  and  $\alpha_{41}$  satisfy  $(\gamma - \alpha_1) = \alpha_{41} + \pi = 0, \pm\pi$ , there can be cancellations between  $|\langle m \rangle|_{3-\nu}$  and  $|\langle m \rangle|_4$  and there is no significant lower bound on  $|\langle m \rangle|$ . We get for negligible  $m_1$ ,

$$\begin{aligned} 0 \text{ (0)} &\leq |\langle m \rangle| \leq 7.0 \text{ (14)} \times 10^{-3} \text{ eV}, & \text{LMA MSW solution;} \\ 0 \text{ (0)} &\leq |\langle m \rangle| \leq 1.5 \text{ (3.2)} \times 10^{-3} \text{ eV}, & \text{SMA MSW solution;} \\ 0 \text{ (0)} &\leq |\langle m \rangle| \leq 3.1 \text{ (3.2)} \times 10^{-3} \text{ eV}, & \text{LOW-QVO solution,} \end{aligned} \quad (115)$$

while for  $0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV}$  one finds

$$0 \text{ (0)} \text{ eV} \leq |\langle m \rangle| \leq 0.2 \text{ (0.2)} \text{ eV} \quad (116)$$

for all solutions of the solar neutrino problem, with  $\max(|\langle m \rangle|) \simeq m_1$ . This result was obtained for the allowed ranges of values of  $\theta_\odot$ ,  $\Delta m_\odot^2$ ,  $|U_{e3}|^2$ ,  $\Delta m_{\text{SBL}}^2$  and  $|U_{e4}|^2$ , which were used to get eq. (114) and which are described after eq. (114). In Fig. 11 we show the allowed values of  $|\langle m \rangle|$  for the different solutions of the solar neutrino problem as a function of  $m_1$ . As the figure demonstrates, there exists an upper bound on  $|\langle m \rangle|$ , but not a non-trivial lower one:  $|\langle m \rangle| \cong 0$  eV is not excluded. However, one has  $|\langle m \rangle| \geq 0.01$  eV in a large region of the relevant parameter space, which can be tested in the  $(\beta\beta)_{0\nu}$ -decay experiments of the next generation. Note also that for the neutrino mass spectrum under discussion a value of  $|\langle m \rangle| \gtrsim 0.03$  eV is possible *only if*  $m_1 \gtrsim 0.02$  eV.

If the CP-symmetry is violated, it would be rather difficult to obtain information on the Majorana CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$  even if it were possible to measure all the other parameters entering into the expression for  $|\langle m \rangle|$ , eq. (106), and  $|\langle m \rangle|$  with sufficient accuracy. The measured values of these parameters and of  $|\langle m \rangle|$  would define a hypersurface in the 3-D parameter space of the three CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$ , on which the phases would be constrained to lie. If such hypersurface does not contain any of the points corresponding to the cases of CP-conservation, i.e.,  $\alpha_{21}, \alpha_{31}, \alpha_{41} = 0, \pm\pi$ , one could conclude that the CP-symmetry is violated in the lepton sector. Otherwise, it would not be possible to draw any conclusions concerning the CP-symmetry violation.

There are several physically interesting cases, however, in which the expression for  $|\langle m \rangle|$  simplifies somewhat and includes effectively only two or just one Majorana CP-violating phases: i)  $m_1$  is negligible, ii) SMA MSW solution of the solar neutrino problem, iii)  $|U_{e3}|^2$ , which is limited by the CHOOZ data, is rather small, e.g.,  $|U_{e3}|^2 \lesssim (0.5 - 1.0) \times 10^{-2}$ , and iv) a combination of any two of the above three possibilities.

If  $m_1$  is negligible, the first term in equation (106) would “drop” and only two relative CP-violating phases would be relevant. Once  $\Delta m_\odot^2$ ,  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_{\text{SBL}}^2$  and the relevant neutrino mixing angles are determined experimentally with a sufficient accuracy, the measurement of  $|\langle m \rangle| \neq 0$  would allow to obtain constraints on the phase differences  $\alpha_{32}$  and  $\alpha_{42}$  using the relation:

$$\begin{aligned} |\langle m \rangle|^2 = & (\sqrt{\Delta m_\odot^2} (1 - |U_{e3}|^2) \sin^2 \theta_\odot)^2 + (\sqrt{\Delta m_{\text{atm}}^2} |U_{e3}|^2)^2 + (\sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2)^2 \\ & + 2\sqrt{\Delta m_\odot^2} \sqrt{\Delta m_{\text{atm}}^2} (1 - |U_{e3}|^2) |U_{e3}|^2 \sin^2 \theta_\odot \cos \alpha_{32} \\ & + 2\sqrt{\Delta m_\odot^2} \sqrt{\Delta m_{\text{SBL}}^2} (1 - |U_{e3}|^2) |U_{e4}|^2 \sin^2 \theta_\odot \cos \alpha_{42} \\ & + 2\sqrt{\Delta m_{\text{atm}}^2} \sqrt{\Delta m_{\text{SBL}}^2} |U_{e3}|^2 |U_{e4}|^2 \cos(\alpha_{42} - \alpha_{32}) . \end{aligned} \quad (117)$$

Figs. 12 and 13 illustrate this possibility.

If the mass  $m_1$  is negligible, one would be able to obtain information on the violation of CP-symmetry in the lepton sector utilizing eq. (117) even if the mixing matrix element  $|U_{e3}|^2$  will be shown to be exceedingly small, so that the terms  $\sim |U_{e3}|^2$  give negligible contribution in  $|\langle m \rangle|$ . In this case the data on  $\Delta m_\odot^2$ ,  $\sin^2 \theta_\odot$ ,  $\Delta m_{\text{SBL}}^2$  and  $|U_{e4}|^2$  together with a measured value of  $|\langle m \rangle| \neq 0$  eV could provide information on the Majorana CP-violating phase  $\alpha_{42}$ :

$$4\sqrt{\Delta m_\odot^2} \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2 \sin^2 \theta_\odot \sin^2 \alpha_{42} \simeq (\sqrt{\Delta m_\odot^2} \sin^2 \theta_\odot + \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2)^2 - |\langle m \rangle|^2 . \quad (118)$$

For the SMA MSW solution of the solar neutrino problem we have both  $\sin^2 \theta_\odot \lesssim 2.5 \times 10^{-3}$  and  $\Delta m_\odot^2 \ll \Delta m_{\text{atm}}^2$ . Neglecting terms  $\sim \sin^2 \theta_\odot$  and  $\sim \Delta m_\odot^2 / \Delta m_{\text{atm}}^2$ , we get from eq. (106):

$$\begin{aligned}
|<m>|^2 &\simeq (m_1(1 - |U_{e3}|^2))^2 + \Delta m_{\text{atm}}^2 |U_{e3}|^4 + \Delta m_{\text{SBL}}^2 |U_{e4}|^4 \\
&+ 2m_1 \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} |U_{e3}|^2 (1 - |U_{e3}|^2) \cos \alpha_{31} \\
&+ 2m_1 \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 (1 - |U_{e3}|^2) \cos \alpha_{41} \\
&+ 2\sqrt{m_1^2 + \Delta m_{\text{atm}}^2} \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e3}|^2 |U_{e4}|^2 \cos(\alpha_{31} - \alpha_{41})
\end{aligned} \tag{119}$$

where we took into account the upper limits  $|U_{e3}|^2 < 0.08$  and  $|U_{e4}|^2 < 0.01$ . If the element  $|U_{e3}|$  of the neutrino mixing matrix is rather small, say  $|U_{e3}|^2 \lesssim (0.5 - 1.0) \times 10^{-2}$ , we would have a further simplification of the expression for  $|<m>|$ , as it follows from eq. (106):

$$|<m>| \simeq |m_1 + \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 e^{i\alpha_{41}}|. \tag{120}$$

For  $m_1 \gg \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2$ , i.e., for  $m_1 \gtrsim 0.04$  eV (see eq. (110)), eq. (120) reduces to  $|<m>| \simeq m_1$ , while if  $m_1 \ll \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2$ ,  $|<m>| \simeq \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2 \lesssim 5 \times 10^{-3}$  eV. For the “intermediate” values of  $m_1 \sim \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2$ , the following relation would be valid:

$$4m_1 \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2 \sin^2 \alpha_{41} 2 = (m_1 + \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2)^2 - |<m>|^2 \tag{121}$$

In deriving this relation we have neglected the terms which are either much smaller than  $m_1^2$ , or are smaller than  $\sim 10^{-5}$  eV<sup>2</sup>. Thus, in the case of the SMA MSW solution of the solar neutrino problem, sufficiently small value of  $|U_{e3}|^2$  and for the 3+1B neutrino mass spectrum under discussion, the measurement of  $\Delta m_{\text{SBL}}^2$ ,  $|U_{e4}|^2$  and  $|<m>|$  might allow to obtain combined constraints on the mass of the lightest neutrino,  $m_1$  and on the CP-violating phase  $\alpha_{41}$ .

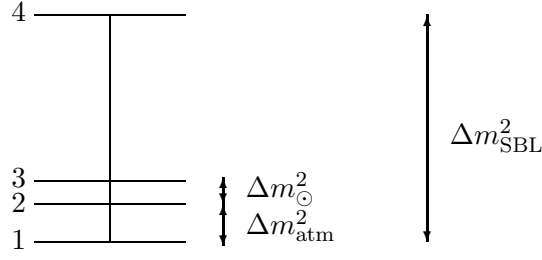
The relation (117), valid for negligible  $m_1$ , simplifies also in the case of the SMA MSW solution of the solar neutrino problem. For this solution the terms  $\sim \sqrt{\Delta m_\odot^2} \sin^2 \theta_\odot$  in eq. (117) can be neglected. In this approximation only one Majorana CP-violating phase “survives” in eq. (117) and we get:

$$\begin{aligned}
&4\sqrt{\Delta m_{\text{atm}}^2} \sqrt{\Delta m_{\text{SBL}}^2} |U_{e3}|^2 |U_{e4}|^2 \sin^2 (\alpha_{42} - \alpha_{32}) 2 \\
&\simeq (\sqrt{\Delta m_{\text{atm}}^2} |U_{e3}|^2 + \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2)^2 - |<m>|^2.
\end{aligned} \tag{122}$$

We have considered so far the cases of 3+1B neutrino mass spectrum, with  $m_1 \ll 0.02$  eV and  $0.02$  eV  $\lesssim m_1 \leq 0.02$  eV, corresponding to mass spectra of  $\nu_{1,2,3}$  of hierarchical type and with partial hierarchy. For  $m_1 \gtrsim 0.3$  eV,  $|<m>|$  is equal, up to corrections which do not exceed  $\sim 5 \times 10^{-3}$  eV, to the effective Majorana mass  $|<m>|_{3-\nu}$  in the case of mixing of 3 quasi-degenerate Majorana neutrinos. Detailed predictions for  $|<m>|_{3-\nu}$  for the latter case were derived in ref. [19] (Section 6). Up to the corrections indicated above, the predictions obtained in [19] are valid also for the spectrum of the 3+1B type and we are not going to reproduce them here. It is interesting to note that for  $m_1 \gtrsim 0.3$  eV, the masses of the three lighter neutrinos coincide up to corrections smaller than 10%:  $m_1 \simeq m_2 \simeq m_3$ . As it is not difficult to convince oneself, under the above conditions we have  $m_{\nu_e} \simeq m_1$ , where  $m_{\nu_e}$  is the mass measured in  $^3\text{H}$   $\beta$ -decay experiments. Therefore, the KATRIN experiment [67] can provide unique information on  $m_1$  if the 3+1B scheme is realized. The measurement of  $m_1 \gtrsim 0.4$  eV will allow to determine, in particular, the values of the masses of the neutrinos  $\nu_{2,3,4}$ , i.e., the neutrino mass spectrum.

## 8 The 3+1C mass spectrum

The 3+1C neutrino mass spectrum differs from the 3+1B spectrum by the role played by  $\Delta m_{21}^2$  and  $\Delta m_{32}^2$ . We have again three nearly-degenerate neutrinos  $\nu_{1,2,3}$  with masses sufficiently smaller than the mass of the fourth one,  $m_1 \simeq m_2 \simeq m_3 < (\ll) m_4$ , as is shown graphically below:



This mass pattern corresponds to

$$\Delta m_{32}^2 \ll \Delta m_{21}^2 \ll \Delta m_{41}^2. \quad (123)$$

Equation (123) leads to the identification:

$$\Delta m_{32}^2 \equiv \Delta m_{\odot}^2, \quad \Delta m_{31}^2 \equiv \Delta m_{\text{atm}}^2, \quad \Delta m_{41}^2 \equiv \Delta m_{\text{LSND}}^2. \quad (124)$$

The masses of the three lighter neutrinos,  $m_{1,2,3}$ , can form a spectrum of the inverted hierarchy, partial inverted hierarchy, or of quasi-degenerate type (see, e.g., [19]). In the first case one has  $m_1 \ll m_{2,3} \simeq \sqrt{\Delta m_{\text{atm}}^2}$ , which implies  $m_1 \ll 0.02$  eV; in the second one has  $0.02$  eV  $\lesssim m_1 \leq 0.2$  eV, while the third corresponds to  $m_1 > 0.2$  eV [19]. We will comment on the predictions for  $|\langle m \rangle|$  when  $m_1 > 0.2$  eV after discussing in detail the other two possibilities.

In accordance with eq. (124) we have:

- i)  $|U_{e1}|^2$  is constrained by the CHOOZ limit, which at 99% C.L. reads [58, 55]  $|U_{e1}|^2 < 0.08$ ;
- ii)  $|U_{e2}|$  and  $|U_{e3}|$  are related to the solar neutrino mixing angle  $\theta_{\odot}$ ,

$$\begin{aligned} |U_{e2}|^2 &= \cos^2 \theta_{\odot} (1 - |U_{e1}|^2), \\ |U_{e3}|^2 &= \sin^2 \theta_{\odot} (1 - |U_{e1}|^2); \end{aligned} \quad (125)$$

- iii)  $|U_{e4}|^2$  is bounded from above and from below by SBL data [35, 51]:  $2 \times 10^{-4} < |U_{e4}|^2 < 1 \times 10^{-2}$ .

Using eq. (124), one obtains for the neutrino masses  $m_{2,3,4}$ :

$$\begin{aligned} m_2 &= \sqrt{m_1^2 - \Delta m_{\odot}^2 + \Delta m_{\text{atm}}^2}; \\ m_3 &= \sqrt{m_1^2 + \Delta m_{\text{atm}}^2}; \\ m_4 &= \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}. \end{aligned} \quad (126)$$

For  $\Delta m_{\odot}^2 \lesssim 2 \times 10^{-4}$  eV<sup>2</sup> we have  $\Delta m_{\odot}^2 / \Delta m_{\text{atm}}^2 \ll 1$ . Correspondingly, the  $\Delta m_{\odot}^2$  correction in  $|\langle m \rangle|$ , which is proportional to  $0.5 \Delta m_{\odot}^2 / \sqrt{m_1^2 + \Delta m_{\text{atm}}^2}$ , does not exceed approximately  $1.1 \times 10^{-3}$  eV and we will neglect it. In this approximation  $|\langle m \rangle|$  does not depend on  $\Delta m_{\odot}^2$ . If values of  $\Delta m_{\odot}^2 \simeq 7 \times 10^{-4}$  eV<sup>2</sup> are allowed by the solar neutrino and CHOOZ data [55],  $\Delta m_{\odot}^2$  in

the expression for  $m_2$  can be non-negligible for values of  $\Delta m_{\text{atm}}^2 \simeq (1.5 - 3.0) \times 10^{-3} \text{ eV}^2$  and  $m_1 \lesssim \Delta m_{\text{atm}}^2$ . For  $m_1 \simeq 0 \text{ eV}$ ,  $\Delta m_{\text{atm}}^2 = 1.5 \times 10^{-3} \text{ eV}^2$  and  $\Delta m_{\odot}^2 = 2 \times 10^{-4}, 7 \times 10^{-4} \text{ eV}^2$ , for instance, we have  $m_2 \simeq 3.6 \times 10^{-2}, 2.8 \times 10^{-2} \text{ eV}$ . For  $m_1^2 \gtrsim \Delta m_{\text{atm}}^2$ , the relative contribution of  $\Delta m_{\odot}^2$  in  $m_2$  is even smaller. Thus, taking the effect due to the presence of  $\Delta m_{\odot}^2$  in the expression for  $m_2$  into account leads to a change by not more than  $\sim 30\%$  in the prediction for the contribution of the term  $\sim m_2$  in  $|\langle m \rangle|$ .

In the case of the neutrino mass spectrum of the 3+1C type under discussion the effective Majorana mass  $|\langle m \rangle|$  is given by:

$$|\langle m \rangle| \simeq |m_1| |U_{e1}|^2 + \sqrt{m_1^2 - \Delta m_{\odot}^2 + \Delta m_{\text{atm}}^2} (1 - |U_{e1}|^2) \cos^2 \theta_{\odot} e^{i\alpha_{21}} + \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} (1 - |U_{e1}|^2) \sin^2 \theta_{\odot} e^{i\alpha_{31}} + \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 e^{i\alpha_{41}}. \quad (127)$$

In the analysis which follows we will neglect  $\Delta m_{\odot}^2$  in the expression for  $m_2$ . In this approximation the expression for  $|\langle m \rangle|$  simplifies somewhat:

$$|\langle m \rangle| \simeq |m_1| |U_{e1}|^2 + \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} (1 - |U_{e1}|^2) (\cos^2 \theta_{\odot} e^{i\alpha_{21}} + \sin^2 \theta_{\odot} e^{i\alpha_{31}}) + \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} e^{i\alpha_{41}} |U_{e4}|^2. \quad (128)$$

The different terms in the r.h.s of eq. (128) for  $|\langle m \rangle|$  can, in general, be of the same order. Thus, all three CP-violating phases  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$  have to be taken into account in the analysis of the allowed ranges of values of  $|\langle m \rangle|$ . This analysis simplifies considerably, however, due to the fact that the absolute value of the sum of the first three contributions in  $|\langle m \rangle|$  due to  $\nu_{1,2,3}$ ,  $|\langle m \rangle|_{3-\nu}$ , is equal to the absolute value of the effective Majorana mass in the case of 3-neutrino mixing and neutrino mass spectrum of inverted hierarchy or of partial inverted hierarchy type. Detailed predictions for  $|\langle m \rangle|_{3-\nu}$  have been obtained recently in ref. [19].

Indeed, one can rewrite  $|\langle m \rangle|$  as follows:

$$|\langle m \rangle| = |\langle m \rangle|_{3-\nu} e^{i(\gamma - \alpha_1)} + \sqrt{\Delta m_{\text{SBL}}^2} |U_{e4}|^2 e^{i\alpha_{41}}, \quad (129)$$

where  $\gamma$  is the overall phase of the sum of the first three terms in eq. (128). The CP-violating phase  $\gamma$  is related to the CP-violating phases  $\alpha_{21}$  and  $\alpha_{31}$  in a complicated way which depends on  $|\langle m \rangle|_1 = m_1 |U_{e1}|^2$ ,  $|\langle m \rangle|_2 = \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} (1 - |U_{e1}|^2) \cos^2 \theta_{\odot}$ , and  $|\langle m \rangle|_3 = \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} (1 - |U_{e1}|^2) \sin^2 \theta_{\odot}$ . Representing  $\langle m \rangle_{3-\nu}$  as a vector in the complex plane, which is a sum of three vectors,  $\langle m \rangle_1$ ,  $\langle m \rangle_2$  and  $\langle m \rangle_3$ , as is sketched in Fig. 1, it becomes clear that the phase  $(\gamma - \alpha_1)$  is equal to the angle formed by the  $\langle m \rangle_1$  (i.e., the  $x$ -axis) and  $\langle m \rangle_{3-\nu}$ . If, e.g., the first term  $|\langle m \rangle|_1$  is negligible, one finds:

$$\gamma - \alpha_1 = \alpha_{21} + \arctan \frac{\sin \alpha_{32}}{\arctan^2 \theta_{\odot} + \cos \alpha_{32}}. \quad (130)$$

In the case of CP-conservation, the phase  $(\gamma - \alpha_1)$  takes into account the overall sign of the term  $\langle m \rangle_{3-\nu}$ .

The allowed intervals of values of  $|\langle m \rangle|_{3-\nu}$  found at 90% C.L. (99% C.L.) in ref. [19] read:

$$0 \text{ (0) eV} \leq |\langle m \rangle|_{3-\nu} \leq 6.8 \text{ (8.1)} \times 10^{-2} \text{ eV}, \quad m_1 \ll 0.02 \text{ eV}; \quad (131)$$

$$0 \text{ (0) eV} \leq |\langle m \rangle|_{3-\nu} \leq 2.1 \text{ (2.2)} \times 10^{-1} \text{ eV}, \quad 0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV}. \quad (132)$$

The upper bounds in eqs. (131) - (132), as can be shown [19], do not depend on  $\theta_{\odot}$  and thus on the solution of the solar neutrino problem. The lower bounds in (124) and (132), however, depend on

$\cos 2\theta_\odot$  and vary with the solution of the solar neutrino problem. The values reported correspond to the LMA MSW and to the LOW-QVO solutions, for which  $\cos 2\theta_\odot = 0$  is possible. For the SMA MSW solution the lower bounds on  $|\langle m \rangle|_{3-\nu}$  read [19]:  $3.7 (3.3) \times 10^{-2}$  eV for the spectrum with inverted hierarchy (or  $m_1 \ll 0.02$  eV), and  $2.8 (1.8) \times 10^{-2}$  eV for the spectrum with partial inverted hierarchy ( $0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV}$ ).

The fourth term  $\sim |U_{e4}|^2$  in eq. (128) can take the following values:

$$3.0 \times 10^{-4} \text{ eV} \leq |\langle m \rangle|_4 \equiv \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |U_{e4}|^2 \leq 5.0 \times 10^{-3} \text{ eV} , \quad (133)$$

where we have used the allowed values of  $|U_{e4}|^2$  and  $\Delta m_{\text{SBL}}^2$ , found at 95% C.L. in ref. [35].

If CP-parity is conserved there exist two possibilities.

**Case A.** If  $\gamma - \alpha_1 = \alpha_{41} = 0, \pm\pi$ , the two contributions in eq. (129) sum up and we have:

$$3.0 \times 10^{-4} \text{ eV} \leq |\langle m \rangle| \leq 6.8 (8.1) \times 10^{-2} \text{ eV} , \quad m_1 \ll 0.02 \text{ eV (negligible } m_1); \quad (134)$$

$$3.0 \times 10^{-4} \text{ eV} \leq |\langle m \rangle| \leq 2.1 (2.2) \times 10^{-1} \text{ eV} , \quad \text{for } 0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV} . \quad (135)$$

The lower limits in these intervals of allowed values are due to the term  $|\langle m \rangle|_4$ , while the maximal values are the same as those in eqs (124) and (132) and correspond to the LMA and LOW-QVO solutions. For the SMA MSW solution they are given in the text after eq. (132).

**Case B.** If  $\gamma - \alpha_1 = \alpha_{41} + \pi = 0, \pm\pi$ , there can be cancellations between the two terms  $|\langle m \rangle|_{3-\nu}$  and  $|\langle m \rangle|_4$ . Correspondingly, there is no non-trivial lower bound on the predicted values of  $|\langle m \rangle|$ . The maximal allowed values of  $|\langle m \rangle|$  are identical to those found in the case A:

$$0 \leq |\langle m \rangle| \leq 6.8 (8.1) \times 10^{-2} \text{ eV} , \quad m_1 \ll 0.02 \text{ eV (negligible } m_1); \quad (136)$$

$$0 \leq |\langle m \rangle| \leq 2.1 (2.2) \times 10^{-1} \text{ eV} , \quad \text{for } 0.02 \text{ eV} \lesssim m_1 \leq 0.2 \text{ eV} . \quad (137)$$

In Fig. 14 we show the dependence of  $|\langle m \rangle|$  on  $\cos 2\theta_\odot$  for the two limiting cases of values of  $m_1$ , i.e. for  $m_1 = 0$  and  $m_1 = 0.2 \text{ eV}$ . As Fig. 14 illustrates, in a large region of the parameter space one can have values of  $|\langle m \rangle| \geq 0.01 \text{ eV}$ , which can be tested in the next generation of the  $(\beta\beta)_{0\nu}$ -decay experiments.

In the case of CP-violation, three CP-violating phases play a role in the determination of  $|\langle m \rangle|$  and therefore it is rather difficult to obtain significant constraints on their values. However, such constraints might be obtained if the term  $m_1|U_{e1}|^2$  gives a negligible contribution in  $|\langle m \rangle|$  and/or in the case of the SMA MSW solution of the solar neutrino problem. If, for instance,  $|U_{e1}|^2$  turns out to be relatively small, say  $|U_{e1}|^2 \lesssim 0.01$ , for  $m_1 \leq 0.2 \text{ eV}$  one could neglect the term  $m_1|U_{e1}|^2$  in eq. (128). Under this condition (i.e., up to corrections  $\sim 2 \times 10^{-3} \text{ eV}$ ) there will be just three terms contributing to  $|\langle m \rangle|$  and two relevant CP-violating phase differences,  $\alpha_{32}$  and  $\alpha_{42}$ . If the other parameters in eq. (128) are known with sufficient accuracy, a measurement of  $|\langle m \rangle| \neq 0$  would allow to constraint the values of  $\alpha_{32}$  and  $\alpha_{42}$ , which have to satisfy:

$$\begin{aligned} & |\langle m \rangle|^2 - (m_1^2 + \Delta m_{\text{atm}}^2) (1 - 12 \sin^2 2\theta_\odot) - \Delta m_{\text{SBL}}^2 |U_{e4}|^4 \\ &= 12(m_1^2 + \Delta m_{\text{atm}}^2) \sin^2 2\theta_\odot \cos \alpha_{32} \\ &+ 2\sqrt{(m_1^2 + \Delta m_{\text{atm}}^2)(m_1^2 + \Delta m_{\text{SBL}}^2)} |U_{e4}|^2 (\cos^2 \theta_\odot \cos \alpha_{42} + \sin^2 \theta_\odot \cos(\alpha_{42} - \alpha_{32})). \end{aligned} \quad (138)$$

To illustrate the correlation between the values of  $(\alpha_4 - \alpha_2)$  and  $(\alpha_3 - \alpha_2)$  eq. (138) would imply, we show in Fig. 15 the phase  $(\alpha_4 - \alpha_2)$  versus  $(\alpha_3 - \alpha_2)$  for a number of fixed values of  $|\langle m \rangle|$  and for two different sets of values of the other parameters entering into eq. (138).

Let us note that in the case of the SMA MSW solution of the solar neutrino problem the terms  $\sim \sin^2 \theta_\odot$  can be neglected in eq. (138). Taking this into account one finds the relation:

$$4\sqrt{(m_1^2 + \Delta m_{\text{atm}}^2)(m_1^2 + \Delta m_{\text{SBL}}^2)} |U_{e4}|^2 \sin^2 \alpha_{42} \simeq (\sqrt{m_1^2 + \Delta m_{\text{atm}}^2} + \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2})^2 - |\langle m \rangle|^2. \quad (139)$$

This relation can be used, in principle, to obtain combined constraints on  $m_1$  and  $\alpha_{42}$ .

We have assumed so far that either  $m_1 \ll 0.02$  eV or  $0.02$  eV  $\lesssim m_1 \leq 0.2$  eV, i.e., that the mass spectrum of the three lighter neutrinos  $\nu_{1,2,3}$  is either of the inverted hierarchy or of partial inverted hierarchy type. If  $m_1 \gtrsim 0.3$  eV, the mass spectrum of  $\nu_{1,2,3}$  will be of quasi-degenerate type. In this case, up to corrections which are smaller than  $\sim 5 \times 10^{-3}$  eV, the spectra 3+1B and 3+1C are indistinguishable in what regards the predictions for  $|\langle m \rangle|$ <sup>12</sup>. Correspondingly, the comments and conclusions made for the 3+1B spectrum at the end of Section 7 concerning  $|\langle m \rangle|$  in the case when  $m_1 \gtrsim 0.3$  eV and the fact that in this case  $m_{\nu_e} \simeq m_1$ , are valid also for the 3+1C spectrum.

## 9 Conclusions.

Assuming mixing of four massive Majorana neutrinos, we have investigated in detail the implications of the solar, atmospheric, LSND and other neutrino oscillation data for the predictions of the possible values of effective Majorana mass,  $|\langle m \rangle|$ , which controls the  $(\beta\beta)_{0\nu}$ -decay rate. Four-neutrino mixing allows to explain the existing evidences and indications for neutrino oscillations, obtained in the solar, atmospheric and LSND experiments, which require three different neutrino mass-squared differences,  $\Delta m_\odot^2$ ,  $\Delta m_{\text{atm}}^2$  and  $\Delta m_{\text{SBL}}^2$ . This is the minimal scheme which describes all available neutrino oscillation data. The fourth neutrino involved in the mixing must be [15, 37] a sterile neutrino,  $\nu_s$  [36]. The sterile neutrino can be accommodated, e.g., in extensions of the Standard Theory, which include  $SU(2)_L$  singlet right-handed (RH) neutrino field(s) (see, e.g., [37]). We have considered all different types of neutrino mass spectrum, which are compatible with the neutrino oscillation data and with the data from  $^3\text{H}$   $\beta$ -decay direct neutrino mass measurements [43, 44, 45, 46, 47, 48, 49]: 2+2A, 2+2B, 3+1A, 3+1B and 3+1C. The results of the neutrino oscillation fits of the latest solar and atmospheric neutrino data [2, 8, 41, 42, 53, 55], and of the data from the LSND [35], KARMEN [50] and BUGEY [51] experiments, have been used in our analysis. The constraints on the neutrino oscillation parameters obtained in the CHOOZ [58] and Palo Verde [59] experiments have also been utilized.

All through the study we have assumed that the  $(\beta\beta)_{0\nu}$ -decay is induced *only* by the (V-A) charged current weak interaction via the exchange of virtual massive Majorana neutrinos, transforming the initial nucleus into the final state nucleus and two free electrons. In the case of mixing involving four massive Majorana neutrinos, the  $(\beta\beta)_{0\nu}$ -decay effective Majorana mass  $|\langle m \rangle|$  depends, in general, on the lightest neutrino mass  $m_1$ , on  $\Delta m_\odot^2$ ,  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_{\text{SBL}}^2$ , on the mixing angle  $\theta_\odot$  characterizing the transitions of the solar neutrinos, and on three physical (Majorana) CP-violating phases,  $\alpha_{21}$ ,  $\alpha_{31}$  and  $\alpha_{41}$ . In the 2+2A and 2+2B schemes,  $|\langle m \rangle|$  depends indirectly also on the parameter  $\cos^2 \beta \leq 1$ , which determines the fraction of the solar  $\nu_e$  and of the atmospheric  $\nu_\mu$  ( $\bar{\nu}_\mu$ ), which can oscillate into the sterile neutrino  $\nu_s$  [41, 42]. However, for most of the neutrino mass spectra of interest the expression for  $|\langle m \rangle|$ , to a good approximation, takes rather simple forms, depending only on few of the indicated physical quantities.

<sup>12</sup>The two expressions for  $|\langle m \rangle| \simeq |\langle m \rangle|_{3-\nu}$  differ only by the two Majorana CP-violating phases present in  $|\langle m \rangle|$  and multiplying terms in  $|\langle m \rangle|$ , which, apart from the phase factors, are identical for the two spectra: these phases are, e.g.,  $\alpha_{21}$  and  $\alpha_{31}$  if the spectrum is of 3+1B type, and  $(\alpha_{31} - \alpha_{21})$  and  $(-\alpha_{31})$  for the 3+1C spectrum. Let us note that at present there does not exist any experimental information on the phases  $\alpha_{21}$  and  $\alpha_{31}$ .



For each of the five types of neutrino mass spectrum considered we have derived detailed predictions for the values of  $|\langle m \rangle|$  for the three solutions of the solar neutrino problem, favored by the current solar neutrino data: the LMA MSW, the SMA MSW and the LOW-QVO one. In each of these cases we have identified the “just-CP-violation” region of values of  $|\langle m \rangle|$  whenever it existed: a value of  $|\langle m \rangle|$  in this region would unambiguously signal the presence of CP-violation in the lepton sector. Analyzing the case of CP-conservation, we have derived predictions for  $|\langle m \rangle|$  corresponding to all possible sets of values of the relative CP-parities of the massive Majorana neutrinos. We have investigated the possibility of cancellation between the different terms contributing to  $|\langle m \rangle|$ . The cases when such a cancellation is impossible and there exists a non-trivial lower bound on  $|\langle m \rangle|$  were identified and the corresponding lower bounds were given.

We have also analyzed in detail the question of whether a measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV in the next generation of  $(\beta\beta)_{0\nu}$ -decay experiments [31, 32, 34, 33] combined with the data from the solar, atmospheric, reactor and accelerator neutrino oscillation experiments and from the future  $^3\text{H}$   $\beta$ -decay experiment KATRIN [67] would allow, and under what conditions, i) to determine the absolute values of the neutrino masses and thus the neutrino mass spectrum, and ii) to establish the existence of CP-violation in the lepton sector. We have pointed out, in particular, that in certain cases of 4-neutrino mass spectra (2+2A, 3+1A) there exists a direct relation between the effective Majorana mass  $|\langle m \rangle|$  and the neutrino mass measured in  $^3\text{H}$   $\beta$ -decay,  $m_{\nu_e}$ , i.e.,  $|\langle m \rangle|$  can be proportional to  $m_{\nu_e}$  and we can have even  $|\langle m \rangle| \simeq m_{\nu_e}$ , and that the measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV [32, 34, 33] and of  $m_{\nu_e} \gtrsim 0.4$  eV [67] will give in these cases the unique possibility to determine the absolute values of all four neutrino masses and to obtain information on CP-violation in the lepton sector.

More specifically, we have found that if the neutrino mass spectrum is of the 2+2A type (with  $m_4^2 - m_3^2 = \Delta m_\odot^2$ , see Section 4), or of 3+1A type (with  $m_2 \simeq m_3 \simeq m_4 > m_1$ , see Section 6), one has  $m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} \gtrsim 0.4$  eV, where the lower bound is determined by the minimal value of  $\Delta m_{\text{SBL}}^2$ , allowed by the LSND data with KARMEN and BUGEY limits taken into account. Thus, we find that for any value of  $m_1$ , the neutrino mass  $m_{\nu_e}$  is predicted to lie in the range planned to be probed by the future Karlsruhe-Mainz-Troitsk experiment KATRIN [67]. Therefore the realization of the KATRIN project will allow to test directly the possibility of 2+2A and 3+1A types of neutrino mass spectrum. A measurement of  $m_{\nu_e} \gtrsim 0.4$  eV and a more accurate knowledge of  $\Delta m_{\text{SBL}}^2$  would permit to determine the value of  $m_1$ . This would allow to determine also the values of  $m_{2,3,4}$ .

In the 2+2A scheme, up to corrections  $\sim 10^{-3}$  eV only the two heavier Majorana neutrinos  $\nu_{3,4}$  make contributions to  $|\langle m \rangle|$ . The effective Majorana mass  $|\langle m \rangle|$  depends on one CP-violating phase,  $\alpha_{34} = \alpha_{31} - \alpha_{41}$ . For the SMA solution of the solar neutrino problem one has independently of whether CP is conserved or not,  $|\langle m \rangle| \simeq m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} \gtrsim 0.4$  eV. The same result is valid for the LMA and the LOW-QVO solutions in the case of CP-conservation if  $\phi_3 = \phi_4$ , where  $i\phi_{3,4}$  are the CP-parities of the Majorana neutrinos  $\nu_{3,4}$ . For  $\phi_3 = -\phi_4$  we find  $|\langle m \rangle| \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} |\cos 2\theta_\odot| \simeq m_{\nu_e} |\cos 2\theta_\odot|$  and in most of the allowed region of the relevant parameter space one has  $|\langle m \rangle| \gtrsim 0.01$  eV. The measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV,  $m_{\nu_e} \gtrsim 0.4$  eV, and a more precise determination of  $\cos 2\theta_\odot$  would allow in the case of the LMA MSW or LOW-QVO solutions of the solar neutrino problem and neutrino mass spectrum of 2+2A type to establish whether CP-symmetry is violated in the lepton sector, and will permit to determine the relative CP-parity of the neutrinos  $\nu_{3,4}$  if one of the CP-conservation cases indicated above will be found to be realized. There exists, in particular, a “just-CP-violation” region of values of  $|\langle m \rangle|$ : a measured value of  $|\langle m \rangle|$  in this region would imply that the CP-symmetry is violated in the lepton sector. Some of our results for the 2+2A spectrum are illustrated in Figs. (7) - (10).

In the case of neutrino mass spectrum of the 2+2B type ( $m_4^2 - m_3^2 = \Delta m_{\text{atm}}^2$ ), discussed in

Section 5, *one can have*  $|\langle m \rangle| \gtrsim 3.0 \times 10^{-2} \text{ eV}$  *only if*  $m_1 \gtrsim 10^{-2} \text{ eV}$ : for  $m_1 \leq 10^{-3} \text{ eV}$ , for instance, we find  $|\langle m \rangle| \lesssim 2.0 \times 10^{-2} \text{ eV}$  for the LMA MSW solution, and  $|\langle m \rangle| \lesssim 6.0 \times 10^{-3} \text{ eV}$  for the SMA MSW and LOW-QVO solutions. As  $m_1$  increases, the maximal allowed values of  $|\langle m \rangle|$  also increase, and for  $m_1 \simeq 0.10 \text{ eV}$  we get  $\max(|\langle m \rangle|) \simeq 0.10 \text{ eV}$  for all solutions of the solar neutrino problem. A measured value of  $|\langle m \rangle| \gtrsim 0.03 \text{ eV}$  in the case of the 2+2B neutrino mass spectrum would imply a lower limit on the mass  $m_1$ ,  $m_1 \gtrsim 10^{-2} \text{ eV}$ ; and it would essentially determine the value of  $m_1$  if the SMA MSW solution of the solar neutrino problem turns out to be the correct one since in this case  $|\langle m \rangle| \simeq m_1$ . These results are illustrated in Fig. 6. For  $m_1 \lesssim 10^{-2} \text{ eV}$ , we have  $|\langle m \rangle| \lesssim 3.0 \times 10^{-2} \text{ eV}$ , three CP-violating phases enter into the expression for  $|\langle m \rangle|$  and the CP-violation analysis in this case would be rather complicated and hardly conclusive. If  $m_1 \gtrsim 0.10 \text{ eV}$  and it is found that  $|\langle m \rangle| \gtrsim 2 \times 10^{-2} \text{ eV}$ , however, up to corrections  $\sim 5.0 \times 10^{-3} \text{ eV}$  only one CP-violating phase,  $\alpha_{21}$ , is relevant for the determination of  $|\langle m \rangle|$ :  $|\langle m \rangle| \simeq m_1 |\cos^2 \theta_\odot + e^{i\alpha_{21}} \sin^2 \theta_\odot|$ . Correspondingly, one has  $m_1 |\cos 2\theta_\odot| \lesssim |\langle m \rangle| \lesssim m_1$ ; in the case of the SMA solution  $\sin^2 \theta_\odot \lesssim 2 \times 10^{-3}$  and  $|\langle m \rangle| \simeq m_1$ .

As we have already indicated above, the 3+1A scheme can be tested in the future  $^3\text{H}$   $\beta$ -decay experiment KATRIN [67]. In this scheme  $|\langle m \rangle| \sim m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} \gtrsim 0.4 \text{ eV}$ . In a large region of the relevant parameter space one has  $|\langle m \rangle| \gtrsim 0.01 \text{ eV}$ , and even  $|\langle m \rangle| \gtrsim 0.10 \text{ eV}$ , which can be tested in the current and future  $(\beta\beta)_{0\nu}$ -decay experiments. Up to corrections  $\sim 10^{-2} m_1 \leq 10^{-2} \text{ eV}$ ,  $|\langle m \rangle|$  depends on  $m_{\nu_e} \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2}$ ,  $\theta_\odot$ , on the element  $|U_{e2}|$  of the neutrino mixing matrix, which is constrained by the CHOOZ and Palo Verde results, and on two CP-violating phases,  $\alpha_{32}$  and  $\alpha_{42}$ . There exists “just-CP-violating” region of values of  $|\langle m \rangle|$ : this region can be spanned by the values of  $|\langle m \rangle|$  only if the CP-symmetry is violated. A sufficiently accurate measurement of  $|\langle m \rangle| \gtrsim 0.01 \text{ eV}$  and of  $m_{\nu_e} \gtrsim 0.4 \text{ eV}$ , as well as of  $\Delta m_{\text{SBL}}^2$ ,  $\theta_\odot$  and  $|U_{e2}| \neq 0$ , will allow to get information on the CP-violation in the lepton sector, or on the relative CP-parities of the massive Majorana neutrinos if the relevant CP-violating phases take their CP-conserving values. The CP-violation analysis simplifies considerably if  $|U_{e2}|^2$  is negligible as well as in the case of the SMA MSW solution of the solar neutrino problem. If both  $|U_{e2}|^2 \lesssim 5 \times 10^{-3}$  and the SMA MSW solution is realized, the measurement of  $|\langle m \rangle| \gtrsim 10^{-2} \text{ eV}$  will hardly provide any information on the violation of the CP-symmetry in the lepton sector since  $|\langle m \rangle| \simeq \sqrt{m_1^2 + \Delta m_{\text{SBL}}^2} (1 + O(10^{-2})) \simeq m_{\nu_e}$ . Our results for the 3+1A spectrum are illustrated in Figs. 7 - 10.

For the 3+1B neutrino mass spectrum ( $m_1 \simeq m_2 \simeq m_3 < (\ll) m_4$ ,  $\Delta m_\odot^2 = \Delta m_{21}^2$ ,  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$ , see Section 7), the effective Majorana mass  $|\langle m \rangle|$  coincides, up to corrections  $\sim 5 \times 10^{-3} \text{ eV}$  with the effective Majorana mass in the 3-neutrino mixing case  $|\langle m \rangle|_{3-\nu}$  and 3-neutrino mass spectrum with hierarchy, or with partial mass hierarchy, or of quasi-degenerate type. Detailed predictions for  $|\langle m \rangle|_{3-\nu}$  in the indicated cases, most of which are valid for the 3+1B spectrum as well, have been derived recently in ref. [19] (Sections 4, 6 and 7). We have found, in particular, that *one can have*  $|\langle m \rangle| \gtrsim 3 \times 10^{-2} \text{ eV}$  *only if*  $m_1 \gtrsim 2 \times 10^{-2} \text{ eV}$ . A measured value of  $|\langle m \rangle| \gtrsim 4 \times 10^{-2} \text{ eV}$  would imply for the 3+1B spectrum:  $|\langle m \rangle| \simeq m_1 (1 - |U_{e3}|^2) (1 - \sin^2 2\theta_\odot \sin^2(\alpha_{21}/2))$ , where  $|U_{e3}|^2 < 0.08$  is constrained by the CHOOZ and Palo Verde data. For the SMA MSW solution of the solar neutrino problem ( $\sin^2 2\theta_\odot \ll 1$ ) and/or for  $\alpha_{21} \simeq 2\pi k$ ,  $k = 0, 1, \dots$ , the above relation offers the possibility to determine the mass of the lightest neutrino  $m_1$ . Information on  $m_1$  can be obtained in the future  $^3\text{H}$   $\beta$ -decay experiment KATRIN [67]: in the 3+1B scheme one has  $m_{\nu_e} \simeq m_1$ . A measurement of  $m_{\nu_e} \gtrsim 0.4 \text{ eV}$  would allow to determine  $m_{2,3,4}$ . Combined with a measurement of  $|\langle m \rangle| \gtrsim 4 \times 10^{-2} \text{ eV}$  in the case of the LMA or LOW-QVO solution and a better determination of  $|U_{e3}|^2$  would allow to obtain information on the CP-violation in the lepton sector. For  $|\langle m \rangle| \lesssim 3 \times 10^{-2} \text{ eV}$  the CP-violation analysis is rather complicated since it would involve three CP-violating phases. It simplifies, however, in several physically interesting cases, when the expression for  $|\langle m \rangle|$  includes effectively only two or just one CP-violating phases: i)

$m_1$  is negligible, ii) SMA MSW solution of the solar neutrino problem, iii)  $|U_{e3}|^2$  is rather small, e.g.,  $|U_{e3}|^2 \lesssim (0.5 - 1.0) \times 10^{-2}$ , and iv) a combination of any two of the above three possibilities. Figures 11 - 13 illustrate some of the results for the 3+1B neutrino mass spectrum.

The 3+1C neutrino mass spectrum (Section 8) differs from the 3+1B spectrum by the role played by  $\Delta m_{21}^2$  and  $\Delta m_{32}^2$ : now  $\Delta m_{\odot}^2 = \Delta m_{32}^2$  and  $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2 \simeq \Delta m_{21}^2$ . The effective Majorana mass  $|\langle m \rangle|$  coincides, up to corrections  $\sim 5 \times 10^{-3}$  eV with the effective Majorana mass in the 3-neutrino mixing case,  $|\langle m \rangle|_{3-\nu}$ , and 3-neutrino mass spectrum with inverted hierarchy, or with partial inverted hierarchy, or of quasi-degenerate type. Detailed predictions for  $|\langle m \rangle|_{3-\nu}$  in the indicated cases have been obtained in ref. [19] (Sections 5, 6 and 7). Most of them are valid for the 3+1C spectrum. As for the other types of neutrino mass spectrum considered, one has  $|\langle m \rangle| \gtrsim 0.01$  eV in a large region of the relevant parameter space. For the SMA MSW solution there exist lower bounds on  $|\langle m \rangle| \simeq |\langle m \rangle|_{3-\nu}$  [19]:  $|\langle m \rangle|_{3-\nu} \gtrsim 3.7$  ( $3.3$ )  $\times 10^{-2}$  eV for the spectrum with inverted hierarchy ( $m_1 \ll 0.02$  eV), and  $|\langle m \rangle|_{3-\nu} \gtrsim 2.8$  ( $1.8$ )  $\times 10^{-2}$  eV for the spectrum with partial inverted hierarchy ( $0.02$  eV  $\lesssim m_1 \leq 0.2$  eV). There are no non-trivial lower bounds on  $|\langle m \rangle| \simeq |\langle m \rangle|_{3-\nu}$  in the case of the LMA and LOW-QVO solutions. The maximal allowed values of  $|\langle m \rangle| \simeq |\langle m \rangle|_{3-\nu}$  for the 3-neutrino spectrum with inverted hierarchy ( $m_1 \ll 0.02$  eV), and for the spectrum with partial inverted hierarchy ( $0.02$  eV  $\lesssim m_1 \leq 0.2$  eV) read, respectively,  $6.8$  ( $8.1$ )  $\times 10^{-2}$  eV and  $2.1$  ( $2.2$ )  $\times 10^{-1}$  eV and are the same for all solutions of the solar neutrino problem. In the case of CP-violation, three CP-violating phases play a role in the determination of  $|\langle m \rangle|$  and therefore it is rather difficult to obtain significant constraints on their values from the measurement of  $|\langle m \rangle| \gtrsim 0.01$  eV. However, such constraints might be obtained if the term  $m_1|U_{e1}|^2$ , where  $|U_{e1}|^2 < 0.08$  is limited by the CHOOZ data, gives a negligible contribution in  $|\langle m \rangle|$  and/or in the case of the SMA MSW solution of the solar neutrino problem. Our results for the 3+1C spectrum are shown in Figs. 14 - 15.

The 3+1B and 3+1C neutrino mass spectra are essentially indistinguishable in what regards the predictions for  $|\langle m \rangle|$  if the lightest neutrino mass satisfies  $m_1 > 0.2$  eV, i.e., if the three lighter neutrinos  $\nu_{1,2,3}$  are quasi-degenerate in mass. Up to corrections  $\sim 5 \times 10^{-3}$  eV,  $|\langle m \rangle|$  is equal to the effective Majorana mass  $|\langle m \rangle|_{3-\nu}$  in the case of mixing of 3 quasi-degenerate Majorana neutrinos. Predictions for  $|\langle m \rangle|_{3-\nu}$  in the indicated case were given in ref. [19] (see Section 6 in [19]). For  $m_1 \gtrsim 0.3$  eV we have (up to corrections smaller than 10%)  $m_1 \simeq m_2 \simeq m_3$  and, correspondingly,  $m_{\nu_e} \simeq m_1$ . Thus, the future  $^3\text{H}$   $\beta$ -decay experiment KATRIN [67] can provide information on  $m_1$ . This can allow to determine the neutrino mass spectrum.

Finally, in Figs. (16), (17) and (18) we show the possible magnitude of  $|\langle m \rangle|$  as a function of  $m_1$  for  $m_1$  varying continuously from  $10^{-5}$  eV to 1.0 eV for the five types of neutrino mass spectra 2+2A,B and 3+1A,B,C compatible with the data and respectively for the LMA MSW, LOW-QVO and SMA MSW solutions of the solar neutrino problem<sup>13</sup>.

To conclude, as in the earlier study [19], we have found that the observation of the  $(\beta\beta)_{0\nu}$ -decay with a rate corresponding to  $|\langle m \rangle| \gtrsim 0.02$  eV, which is in the range of sensitivity of the future  $(\beta\beta)_{0\nu}$ -decay experiments, can provide unique information on the neutrino mass spectrum. Combined with information on the lightest neutrino mass, which could be provided, e.g., by the  $^3\text{H}$   $\beta$ -decay experiment KATRIN [67], it can give also information on the CP-violation in the lepton sector, and if CP-invariance holds - on the relative CP-parities of the massive Majorana neutrinos.

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<sup>13</sup>For the case of mixing of three massive Majorana neutrinos similar plots were proposed also in refs. [26, 27].

## References

- [1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998);
- [2] Super-Kamiokande Collaboration, talk by H. Sobel at *Neutrino 2000* [3], to appear in the Proceedings.
- [3] Homestake Collaboration, talk by K. Lande at *Neutrino 2000*, 19th International Conference on Neutrino Physics and Astrophysics (Sudbury, Canada, 2000), to be published in the Proceedings of the Conference.
- [4] Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996).
- [5] SAGE Collaboration, J.N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999); talk by V. Gavrin at *Neutrino 2000* [3], to be published in the Proceedings.
- [6] GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **447**, 127 (1999).
- [7] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **82**, 2430 (1999).
- [8] Super-Kamiokande Collaboration, talk by Y. Suzuki in *Neutrino 2000* [3], to be published in the Proceedings.
- [9] GNO Collaboration, M. Altmann *et al.*, Phys. Lett. B **490**, 16 (2000); talk by E. Bellotti at *Neutrino 2000* [3], to be published in the Proceedings.
- [10] SNO Collaboration, A. Hallin *et al.*, Nucl. Phys. A **663-664**, 787 (2000).
- [11] J.N. Bahcall, M.H. Pinsonneault, and S. Basu, astro-ph/0010346.
- [12] S. M. Bilenky and B. Pontecorvo, Phys. Rep. **41**, 225 (1978).
- [13] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
- [14] S. T. Petcov, Lecture Notes in Physics, v. **512** (eds. H. Gausterer and C.B. Lang, Springer, 1998), p. 281 (hep-ph/9806466).
- [15] S. M. Bilenky, C. Giunti and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999) (hep-ph/9812360).
- [16] S.T. Petcov, Phys. Lett. B **110**, 245 (1982).
- [17] M. Moe and P. Vogel, Ann. Rev. Nucl. Part. Sci. **44**, 247 (1994); A. Faessler and F. Šimkovic, J. Phys. G **24**, 2139 (1998).
- [18] H. Ejiri, talk at *Neutrino 2000* [3], to appear in the Proceedings.
- [19] S.M. Bilenky, S. Pascoli and S.T. Petcov, Ref. SISSA 1/2001/EP (hep-ph/0102265).
- [20] S. T. Petcov and A. Yu. Smirnov, Phys. Lett. B **322**, 109 (1994).
- [21] S.M. Bilenky, A. Bottino, C. Giunti and C.W. Kim, Phys. Lett. B **356**, 273 (1995) and Phys. Rev. D **54**, 1881 (1996); S.M. Bilenky, C. Giunti, C.W. Kim and M. Monteno, Phys. Rev. D **57**, 6981 (1998).
- [22] S.M. Bilenky, C. Giunti, C.W. Kim and S.T. Petcov, Phys. Rev. D **54**, 4432 (1996).

- [23] F. Vissani, hep-ph/9708483 and JHEP **06**, 022 (1999); H. Minakata and O. Yasuda, Phys. Rev. D **56**, 1692 (1997) and Nucl. Phys. B **523**, 597 (1998); T. Fukuyama, K. Matsuda and H. Nishiura, Phys. Rev. D **57**, 5844 (1998) and hep-ph/9804262; V. Barger and K. Whisnant, Phys. Lett. B **456**, 194 (1999); T. Fukuyama, K. Matsuda, H. Nishiura and N. Takeda, Phys. Rev. **D62**, 93001 (2000), hep-ph/0007237 and hep-ph/0012357.
- [24] S.M. Bilenky, C. Giunti, W. Grimus, B. Kayser and S.T. Petcov, Phys. Lett. B **465**, 193 (1999).
- [25] S.T. Petcov, in *Weak Interactions and Neutrinos*, Proceedings of the 17th International Conference, January 23 - 30, 1999, Cape Town, South Africa (eds. C.A. Dominguez and R.D. Viollier, World Scientific, Singapore, 2000), p. 305; S.M. Bilenky and C. Giunti, *ibid.*, p. 195.
- [26] M. Czakon, J. Gluza and M. Zlarek, hep-ph/0003161; M. Czakon, J. Gluza, J. Studnik and M. Zlarek, hep-ph/0010077.
- [27] H. V. Klapdor-Kleingrothaus, H. Pas and A. Yu. Smirnov, hep-ph/0003219.
- [28] W. Rodejohann, hep-ph/0008044; H. Pas and T.J. Weiler, hep-ph/0101091; D. Falcone and F. Tramontano, hep-ph/0102136.
- [29] L. Baudis *et al.*, Phys. Rev. Lett. **83**, 41 (1999); H. V. Klapdor-Kleingrothaus *et al.*, submitted for publication to Phys. Lett. B and talk given at NOW2000, Otranto, Italy, September 2000 (<http://www.ba.infn.it/~now2000>).
- [30] C.E. Aalseth, F.T. Avignone III *et al.*, Physics of Atomic Nuclei **63**, 1225-1228 (2000).
- [31] NEMO3 proposal, LAL-preprint 94-29 (1994); C. Marquet *et al.*, to appear in Nucl. Phys. B (Proc. Suppl.) **87** (2000).
- [32] E. Fiorini *et al.*, Phys. Rep. **307**, 309 (1998).
- [33] H. V. Klapdor-Kleingrothaus, J. Hellmig and M. Hirsch, J. Phys. G **24**, 483 (1998); L. Baudis *et al.*, Phys. Rep. **307**, 301 (1998).
- [34] M. Danilov *et al.*, Phys. Lett. B **480**, 12 (2000).
- [35] LSND Collaboration, talk by G. Mills at *Neutrino 2000* [3], to appear in the Proceedings.
- [36] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)].
- [37] J.T. Peltoniemi and J.W.F. Valle, Nucl. Phys. B **406**, 409 (1993); D.O. Caldwell and R.N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); E. Ma and P. Roy, *ibid* **52**, R4780 (1995); *ibid* **58**, 071301 (1998); E.J. Chun, C.W. Kim and U.W. Lee, *ibid* **58**, 093003 (1998); K. Benakli and A.Yu. Smirnov, Phys. Rev. Lett. **79**, 4314 (1997); W. Grimus, R. Pfeiffer and T. Schwetz, hep-ph/9905320 [to be published in Eur. Phys. J. C].
- [38] C. Giunti, Phys. Rev. **D61**, 036002 (2000).
- [39] M. Czakon, J. Gluza, J. Studnik and M. Zlarek, Acta. Phys. Polon. B **31**, 1365 (2000).
- [40] A. Kalliomaki and J. Maalampi, Phys. Lett. B **484**, 64 (2000).
- [41] G.L. Fogli, E. Lisi and A. Marrone, Phys. Rev. D **63**, 053008 (2001).
- [42] M.C. Gonzalez-Garcia and C. Peña-Garay, Nucl. Phys. Proc. Suppl. **91**, 80 (2000).
- [43] S.M. Bilenky, C. Giunti and W. Grimus, Phys. Rev. D **57**, 1920 (1998).

- [44] S. Goswami, *ibid* Phys. Rev. D **55**, 2931 (1997).
- [45] V. Barger, T.J. Weiler and K. Whisnant, Phys. Lett. B **427**, 97 (1998); V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Rev. D **58**, 093016 (1998); V. Barger, Y.B. Dai, K. Whisnant and B.L. Young, *ibid* **59**, 113010 (1999).
- [46] V. Barger, B. Kayser, J. Learned, T. Weiler and K. Whisnant, Phys. Lett. B **489**, 345 (2000).
- [47] O.L.G. Peres and A.Yu. Smirnov, Nucl. Phys. B **599**, 3 (2001).
- [48] C. Giunti, hep-ph/0012236.
- [49] W. Grimus and T. Schwetz, hep-ph/0102252.
- [50] KARMEN Collaboration, K. Eitel, Nucl. Phys. Proc. Suppl. **91**, 191 (2000).
- [51] BUGEY Collaboration, Nucl. Phys. B **434**, 503 (1995).
- [52] MiniBooNE Collaboration, talk by A. Bazarko at *Neutrino 2000* [3], to appear in the Proceedings.
- [53] G.L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. **D62**, 113003 (2000).
- [54] P. I. Krastev, talk given at NOW2000, Otranto, Italy, September 2000 (<http://www.ba.infn.it/~now2000>).
- [55] C. Gonzalez-Garcia *et al.*, M. Maltoni, C. Peña-Garay, and J.W.F. Valle, hep-ph/0009350.
- [56] P.I. Krastev and S.T. Petcov Phys. Rev. **D53**, 1665 (1996).
- [57] P.I. Krastev, Q.Y. Liu and S.T. Petcov, Phys. Rev. **D54**, 7057 (1996).
- [58] M. Appolonio *et al.*, Phys. Lett. B **466**, 415 (1999) (hep-ex/9907037).
- [59] F. Boehm, J. Busenitz *et al.*, Phys. Rev. Lett. **84**, 3764 (2000) and Phys. Rev. **D62**, 072002 (2000).
- [60] MINOS Collaboration, P. Adamson *et al.*, NuMI-L-476 (March 1999); Web page: <http://www.-hep.anl.gov/ndk/hypertext/numi.html>.
- [61] K2K Collaboration, Y. Oyama *et al.*, hep-ex/9803014; talk by K. Nakamura at *Neutrino 2000* [3]; K2K Web page: <http://pnahep.kek.jp/>.
- [62] Borexino Collaboration, talk by G. Ranucci at *Neutrino 2000* [3], to appear in the Proceedings.
- [63] KamLAND Collaboration, talk by A. Piepke at *Neutrino 2000* [3], to appear in the Proceedings.
- [64] A. Ereditato, talk presented at the NOW2000, September 2000, Otranto, Italy (<http://www.ba.infn.it/~now2000>).
- [65] V. Lobashov *et al.*, talk at *Neutrino 2000* [3], to appear in the Proceedings.
- [66] C. Weinheimer *et al.*, talk at *Neutrino 2000* [3], to appear in the Proceedings.
- [67] A. Aseev *et al.*, Int. Workshop on Neutrino Masses in the Sub-eV Range, January 18 - 21, Bad Liebenzell, Germany.
- [68] E. Gawiser, astro-ph/0005475.

- [69] MAP collaboration, <http://map.gsfc.gov/>; PLANCK collaboration, <http://astro.estec.esa.ne/SA-general/Projects/Planck>.
- [70] S.M. Bilenky, J. Hošek and S.T. Petcov, Phys. Lett. B **94**, 495 (1980).
- [71] Yu. Kobzarev *et al.*, Yad. Fiz. **32**, 1590 (1980) [Sov. J. Nucl. Phys. **32**, 823 (1980)]; M. Doi *et al.*, Phys. Lett. B **102**, 323 (1981).
- [72] C. Jarskog, Z. Phys. C**29**, 491 (1985); Phys. Rev. **D35**, 1685 (1987).
- [73] J. F. Nieves and P.P. Pal, Phys. Rev. **D36**, 315 (1987).
- [74] J.A. Aguilar-Saavedra and G.C. Branco, Phys. Rev. **D62**, 096009 (2000).
- [75] P.I. Krastev and S.T. Petcov, Phys. Lett. B **205**, 64 (1988).
- [76] P. Langacker *et al.*, Nucl. Phys. B **282**, 589 (1987).
- [77] L. Wolfenstein, Phys. Lett. B **107**, 77 (1981).
- [78] S. M. Bilenky, N. P. Nedelcheva and S. T. Petcov, Nucl. Phys. B **247**, 589 (1984).
- [79] B. Kayser, Phys. Rev. **D30**, 1023 (1984).
- [80] F. Vissani, Nucl. Phys. Proc. Suppl. **100**, 273 (2001).

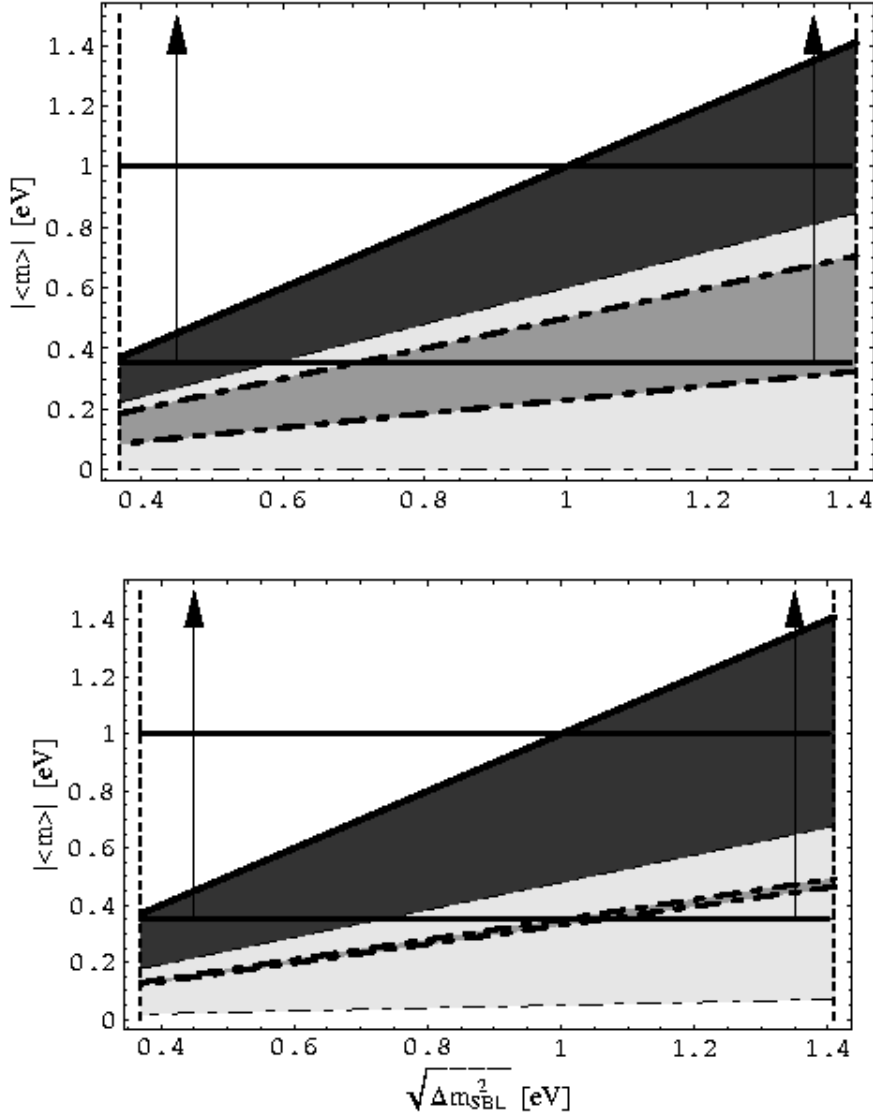


Figure 2: The effective Majorana mass  $|\langle m \rangle|$  as a function of  $\sqrt{\Delta m_{\text{SBL}}^2}$  for the 2+2A neutrino mass spectrum, eq. (39). The allowed regions (in grey) correspond to the LMA solution of ref. [42] for  $\cos^2 \beta = 0.3$  (upper panel) and  $\cos^2 \beta = 0.5$  (lower panel). In the case of CP-invariance and for the 90% (99%) C.L. LMA solution region,  $|\langle m \rangle|$  can have values *i*) for  $\phi_3 = \phi_4$  - on the upper doubly-thick solid line (upper doubly-thick solid line) and *ii*) for  $\phi_3 = -\phi_4$  - in the medium grey (light grey and medium grey) region limited by the thick (thin) dash-dotted lines. If CP is not conserved,  $|\langle m \rangle|$  can lie in any of the regions marked by different grey scales. The dark-grey region corresponds to “just-CP-violation”:  $|\langle m \rangle|$  can have value in this region *only if the CP-parity is not conserved*. The two horizontal (thick) lines show the upper limits [29], quoted in eq. (4).



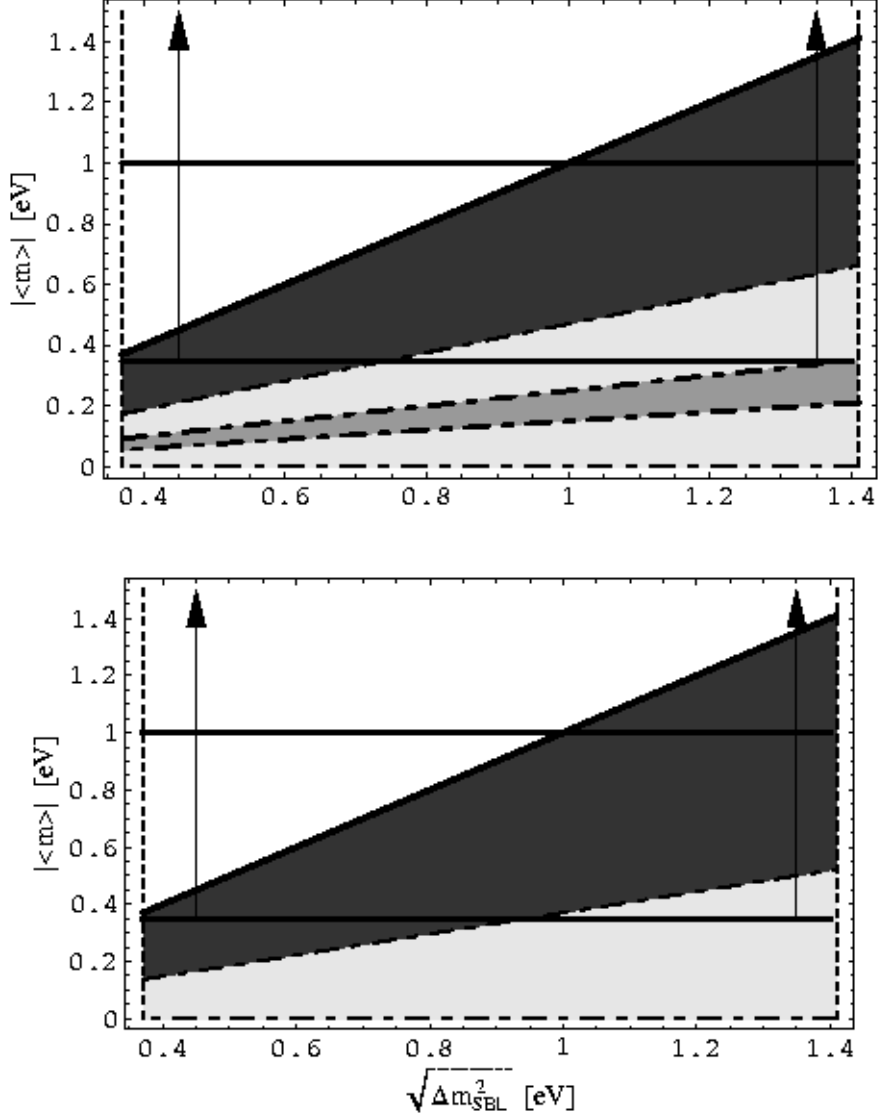


Figure 3: The same as in Fig. (2) for the 90% (99%) C.L. LOW-QVO solution of ref. [42] for  $\cos^2 \beta = 0.3$  (upper panel) and  $\cos^2 \beta = 0.5$  (lower panel). The doubly-thick (doubly-thick) solid lines and the medium grey (light grey and medium grey) lower region, bounded by the thick (thin) dash-dotted lines, correspond to the two cases of CP-invariance,  $\phi_3 = \phi_4$  and  $\phi_3 = -\phi_4$ , respectively. If CP is not conserved,  $|\langle m \rangle|$  can lie in any of the regions marked by different grey scales. The “just-CP-violation” region is shown in dark-grey color:  $|\langle m \rangle|$  can have value in this region *only if the CP-symmetry is violated*.

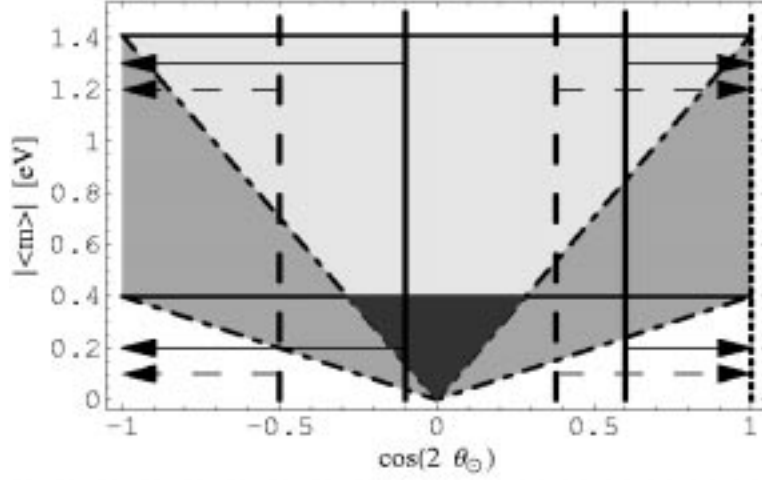


Figure 4: The dependence of  $|\langle m \rangle|$  on  $\cos 2\theta_\odot$  for the 2+2A neutrino mass spectrum, eq. (39). The region between the two thick horizontal solid lines (in light grey and medium grey colors) and the two triangular regions between the thick dash-dotted lines (in medium grey color), correspond to the two CP- conserving cases,  $\phi_3 = \phi_4$  and  $\phi_3 = -\phi_4$ , respectively. The “just-CP-violation” region is denoted by dark-grey color. The regions between each of the two pairs of vertical lines of a given type - doubly thick solid and doubly thick dashed, correspond to the intervals of values of  $\cos 2\theta_\odot$  for the LMA solution and the LOW-QVO solution derived (at 99% C.L.) in ref. [42] for  $\cos^2 \beta = 0.3$  while the vertical dotted line corresponds to the SMA solution.

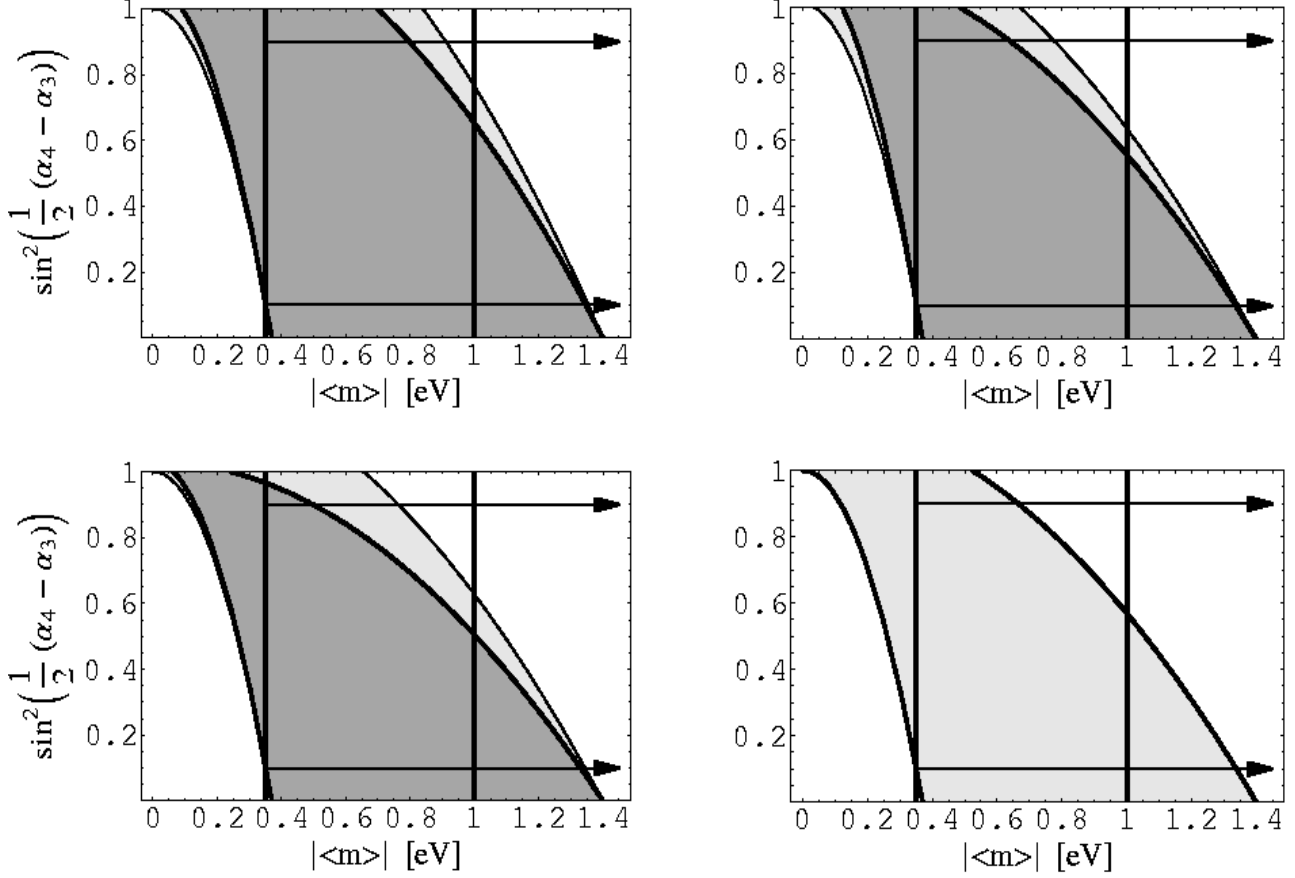


Figure 5: The CP-violation factor  $\sin^2(\alpha_4 - \alpha_3)/2$  as a function of  $|\langle m \rangle|$  in the case of 2+2A mass spectrum, eq. (39), for the LMA (upper panels) and LOW-QVO (lower panels) solutions of ref. [42] and for  $\cos^2 \beta = 0.3$  (upper and lower left panels) and  $\cos^2 \beta = 0.5$  (upper and lower right panels). The results correspond to the 90% C.L. (medium grey region with thick contours) and 99% C.L. (light grey and medium grey region limited by one ordinary solid line and one thick solid line) solution regions. A value of  $\sin^2(\alpha_4 - \alpha_3)/2 \neq 0, 1$ , would signal CP-violation. The two vertical (thick) lines show the upper limits [29], quoted in eq. (4).

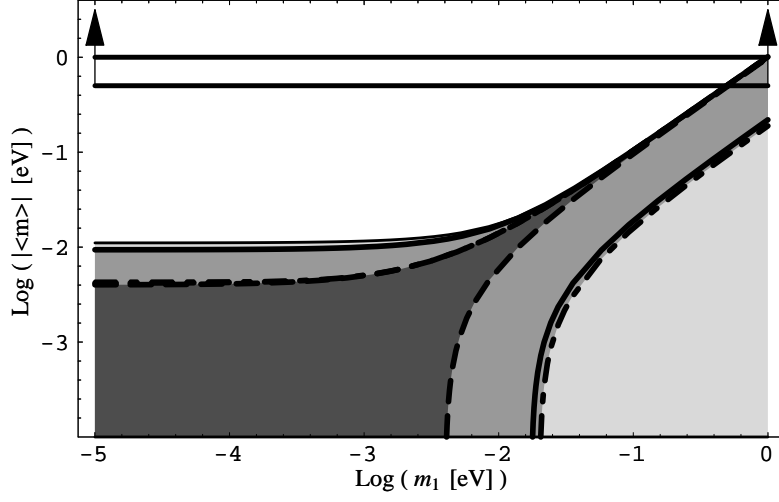


Figure 6: The effective Majorana mass  $|\langle m \rangle|$ , allowed by the data from the solar and CHOOZ experiments, as a function of  $m_1$  for the 2+2B neutrino mass spectrum, eq. (58). The values of  $|\langle m \rangle|$  are obtained for the allowed ranges of  $\Delta m_{\text{SBL}}^2$  and  $|U_{e3}|^2 + |U_{e4}|^2$  derived in ref. [35] at 95% C.L. and for  $\Delta m_{\odot}^2$ ,  $\sin^2 \theta_{\odot}$  from *i*) the 90% C.L. (medium-grey and dark-grey regions limited by the two doubly-thick solid lines and the axes) and the 99% C.L. (all grey regions) LMA MSW solution region, *ii*) the 90% C.L. (medium-grey and dark-grey regions limited by the two doubly-thick dash-dotted lines and the axes) and the 99% C.L. (grey regions below the upper doubly-thick dash-dotted line) LOW-QVO solution regions, and *iii*) the 99% C.L. SMA solution region (dark-grey region limited by the doubly-thick dash-dotted and dashed lines and the axes). The results shown correspond to  $\cos^2 \beta = 0.3$ . The two horizontal lines show the upper limits [29], given in eq. (4).

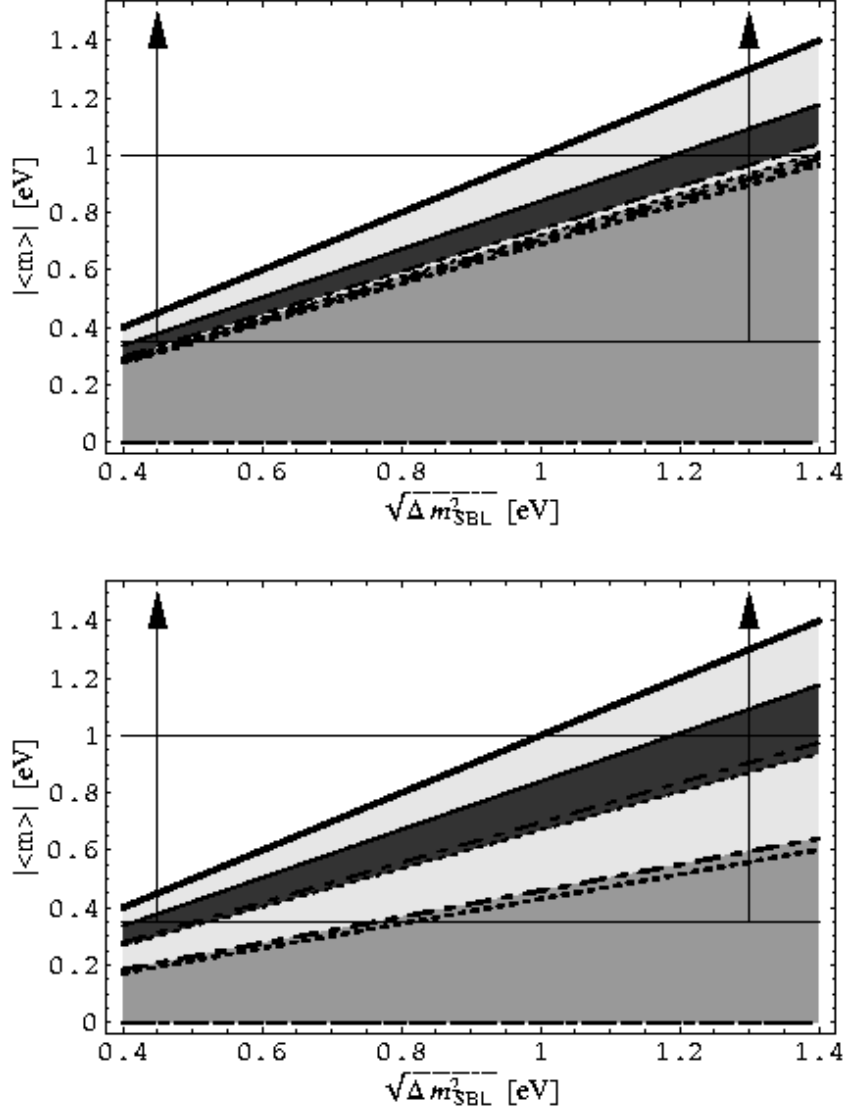


Figure 7:  $|\langle m \rangle|$  as a function of  $\sqrt{\Delta m_{\text{SBL}}^2}$  for the 3+1A neutrino mass spectrum, eq. (79), and the LMA solution (upper panel) and LOW-QVO solution (lower panel) of the  $\nu_\odot$ -problem found in ref. [55] at 90% C.L. (99% C.L.). The regions allowed in the cases of CP-invariance correspond to *i*)  $\phi_3 = \phi_4 = \phi_2$  - the thick solid (non-horizontal) line  $|\langle m \rangle| = \sqrt{\Delta m_{\text{SBL}}^2}$ , *ii*)  $\phi_3 = \phi_4 = -\phi_2$  - the light grey triangular region between the thick and the normal solid lines (light grey triangular region between the thick and the normal solid lines), *iii*)  $\phi_3 = -\phi_4 = \pm\phi_2$  (two cases) - the medium grey triangular regions between the two thick dashed-dotted lines and between the two thick dotted lines (light-grey and medium-grey region between the thin dashed-dotted line and the horizontal axes and between the thin dotted line and the horizontal axes). The “just-CP-violation” region is denoted by dark-grey color. The two horizontal (thick) lines show the upper limits [29], quoted in eq. (4).

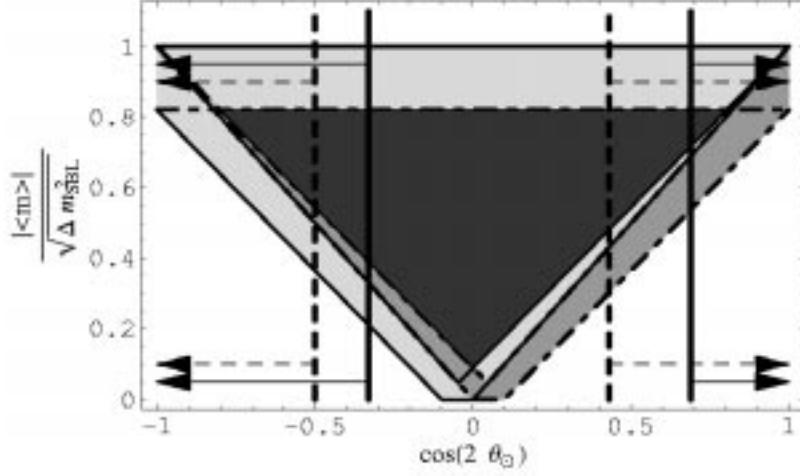


Figure 8: The dependence of  $|\langle m \rangle| / \sqrt{\Delta m_{\text{SBL}}^2}$  on  $\cos 2\theta_{\odot}$  for the 3+1A neutrino mass spectrum, eqs. (79). If CP-invariance holds, the values of  $|\langle m \rangle| / \sqrt{\Delta m_{\text{SBL}}^2}$  lie: i) for  $\phi_3 = \phi_4 = \phi_2$  - on the line  $|\langle m \rangle| / \sqrt{\Delta m_{\text{SBL}}^2} = 1$ , ii) for  $\phi_3 = \phi_4 = -\phi_2$  - in the region between the horizontal thick solid and dash-dotted lines (in light grey and medium grey colors), iii) for  $\phi_3 = -\phi_4 = +\phi_2$  - in the light grey polygon with solid-line contours and iv) for  $\phi_3 = -\phi_4 = -\phi_2$  - in the medium grey polygon with the dash-dotted-line contours. The “just-CP-violation” region is denoted by dark-grey color. The values of  $\cos 2\theta_{\odot}$  between the doubly thick solid and dashed lines correspond to the 99% C.L. LMA and LOW-QVO solution regions of refs. [55], respectively.

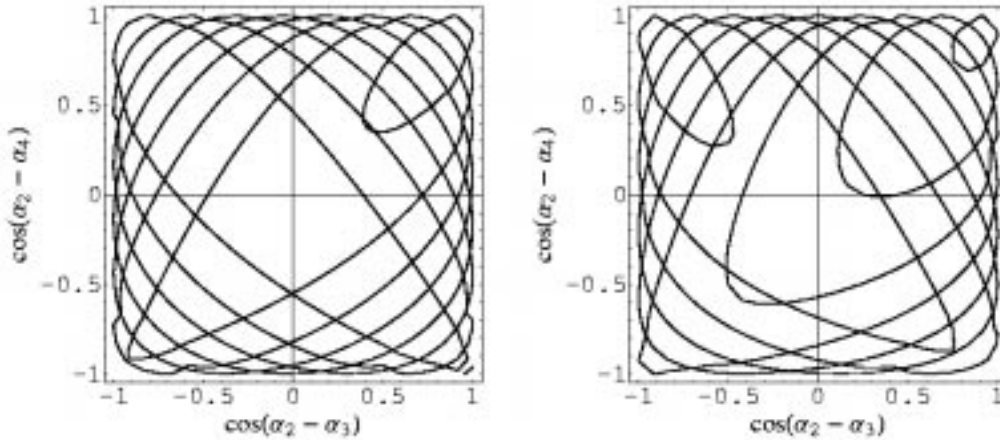


Figure 9: The interdependence of the two CP-violating phases,  $\alpha_{42}$  and  $\alpha_{32}$ , for a given value of the ratio  $|\langle m \rangle| / \sqrt{\Delta m_{\text{SBL}}^2}$  in the case of the 3+1A neutrino mass spectrum. The figures are obtained for  $|\langle m \rangle| = \sqrt{0.15 + 0.10n} \sqrt{\Delta m_{\text{SBL}}^2}$  with  $n = 0, 1 \dots 8$  (with increasing  $|\langle m \rangle|$  from left to right) and  $|U_{e2}|^2 = 0.04$  (left-hand plot) and  $|U_{e2}|^2 = 0.08$  (right-hand plot) and the best fit values of the  $\theta_{\odot}$  parameter from [55] (left-hand plot). The values of  $\cos \alpha_{32,42} = 0, \pm 1$ , correspond to CP-invariance.

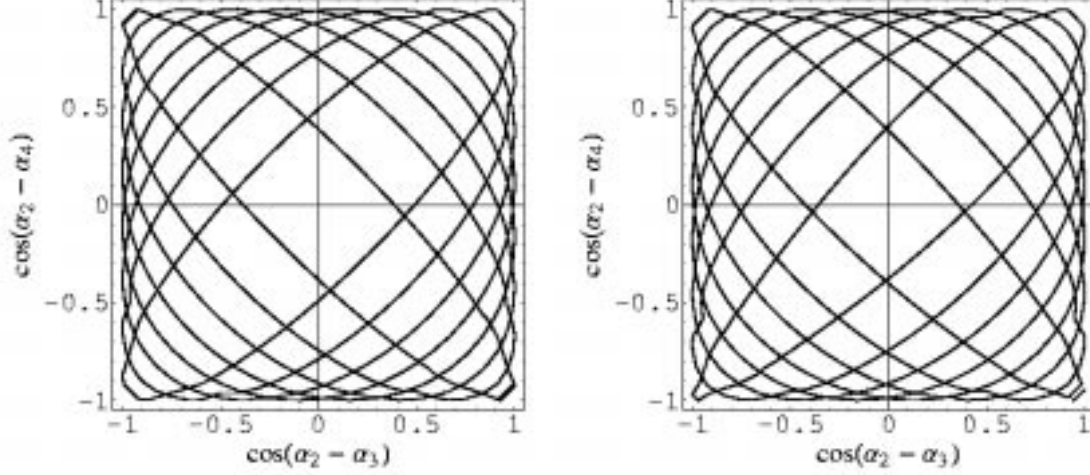


Figure 10: The same as in Fig. 9 for  $|U_{e2}|^2 = 0$  (left-hand plot) and  $|U_{e2}|^2 = 0.01$  (right-hand plot).

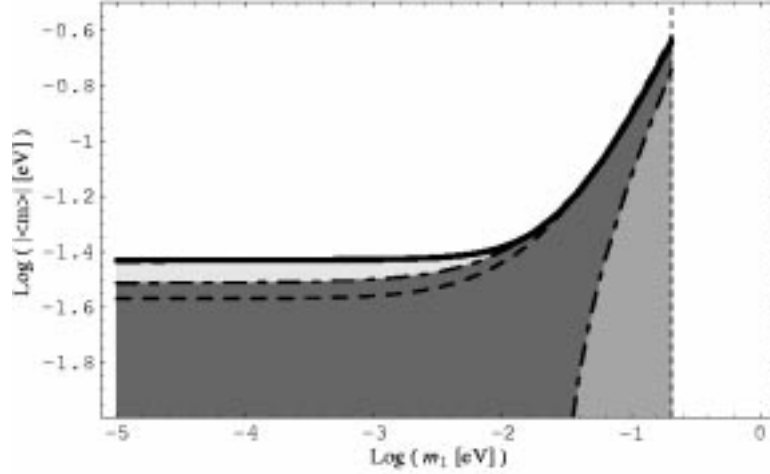


Figure 11: The effective Majorana mass  $|\langle m \rangle|$ , allowed by the data from the solar and CHOOZ experiments, as a function of  $m_1$  in the 3+1B mass spectrum, eq. (102). The values of  $|\langle m \rangle|$  are obtained for the allowed ranges of  $\Delta m_{\text{SBL}}^2$  and  $|U_{e4}|^2$  found in ref. [35] at 95% C.L., for  $\Delta m_{\text{atm}}^2$ ,  $|U_{e3}|^2$  and  $\Delta m_{\odot}^2$ ,  $\sin^2 \theta_{\odot}$  from *i*) the 90% C.L. and 99% C.L. LMA MSW solution regions, (all grey regions), *ii*) the 90% C.L. and 99% C.L. LOW-QVO solution regions (medium and dark grey regions under the thick dashed line) and *iii*) the 99% C.L. SMA MSW solution region (dark grey region among the dash-dotted lines and the axes), derived in ref. [55].

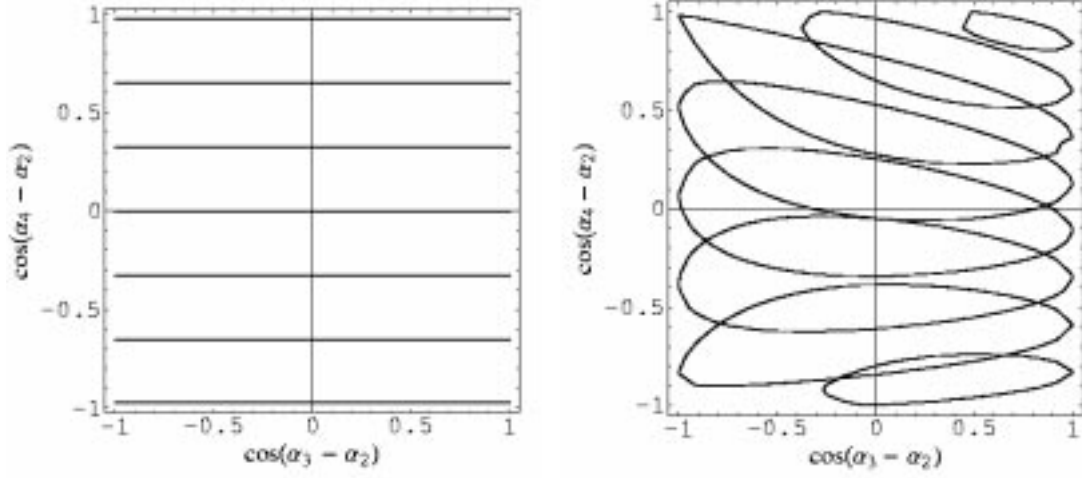


Figure 12: The interdependence of the two CP-violating phases,  $\alpha_{42}$  and  $\alpha_{32}$ , for a given value of  $|\langle m \rangle|$  in the case of the 3+1B neutrino mass spectrum. The figures are obtained for  $|U_{e3}|^2 = 0$  and  $|\langle m \rangle| = \sqrt{0.12 + 0.05n} \times 10^{-2}$  eV with  $n = 0, 1 \dots 6$  (with increasing  $|\langle m \rangle|$  from left to right) (left-hand plot) and for  $|U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2} = 7 \times 10^{-4}$  eV and  $|\langle m \rangle| = \sqrt{7 + 5n} \times 10^{-3}$  eV with  $n = 0, 1 \dots 8$  (with increasing  $|\langle m \rangle|$  from left to right) (right-hand plot) and for the best fit value of  $|U_{e2}|^2 \sqrt{\Delta m_{\odot}^2} = 1.6 \times 10^{-3}$  eV of ref. [55], for the maximum allowed value of  $|U_{e4}|^2 \sqrt{\Delta m_{\text{SBL}}^2} = 5.0 \times 10^{-3}$  eV found in ref. [35]. The values of  $\cos \alpha_{32,42} = 0, \pm 1$ , correspond to CP-invariance.

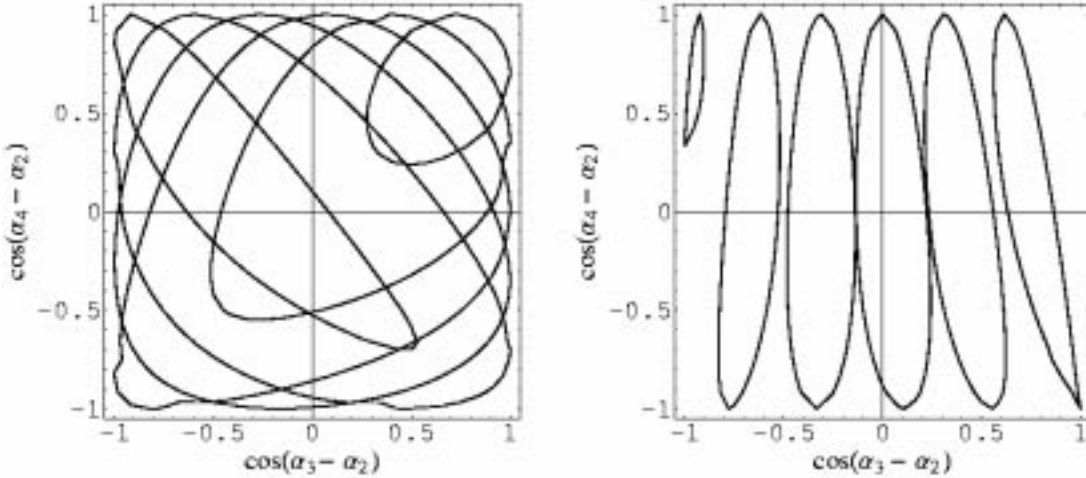


Figure 13: The same as Fig. (12). The figures are obtained for *i*) the maximum allowed value of  $|U_{e4}|^2 \sqrt{\Delta m_{\text{SBL}}^2} = 5.0 \times 10^{-3}$  eV from ref. [35] and  $|\langle m \rangle| = \sqrt{0.1 + 0.3n} \times 10^{-2}$  eV with  $n = 0, 1 \dots 5$  (with increasing  $|\langle m \rangle|$  from left to right) (left-hand plot) and *ii*) the minimum allowed value of  $|U_{e4}|^2 \sqrt{\Delta m_{\text{SBL}}^2} = 3.0 \times 10^{-4}$  eV found in ref. [35] and  $|\langle m \rangle| = \sqrt{0.30 + 0.08n}$  eV with  $n = 0, 1 \dots 5$  (with increasing  $|\langle m \rangle|$  from left to right) (right-hand plot) and *iii*) for the best fit value of  $|U_{e2}|^2 \sqrt{\Delta m_{\odot}^2} = 1.6 \times 10^{-3}$  eV derived in ref. [55], for the maximum allowed value of  $|U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2} = 7.0 \times 10^{-3}$  eV from ref. [55]. The values of  $\cos \alpha_{32,42} = 0, \pm 1$ , correspond to CP-invariance.



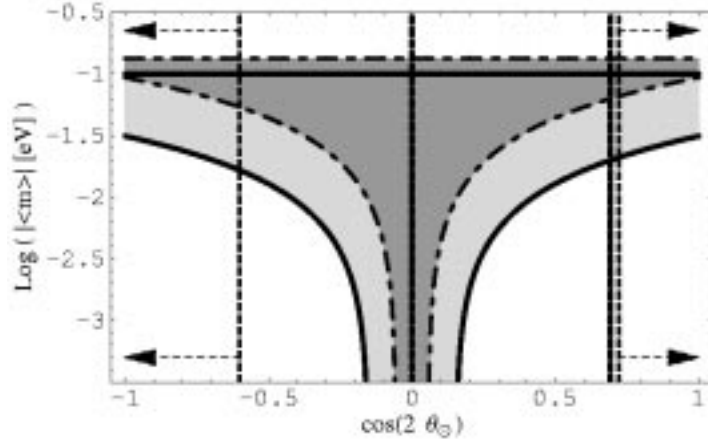


Figure 14: The dependence of  $|\langle m \rangle|$  on  $\cos 2\theta_\odot$  in the case of the 3+1C neutrino mass spectrum, eq. (123), for the 99% C.L. allowed values of the solar and atmospheric neutrino oscillation parameters from ref. [55] and for  $\Delta m_{\text{SBL}}^2$  and  $|U_{e4}|^2$  from ref. [35] at 95% C.L., and  $m_1 = 0.2$  eV (medium grey region with thick dash-dotted contours) and  $m_1 = 0$  eV (medium grey and light grey region with thick solid contours). The values of  $\cos 2\theta_\odot$  between the doubly thick dashed (the thick dashed) lines correspond to the 90% C.L. (99% C.L.) LMA solution regions in ref. [55].

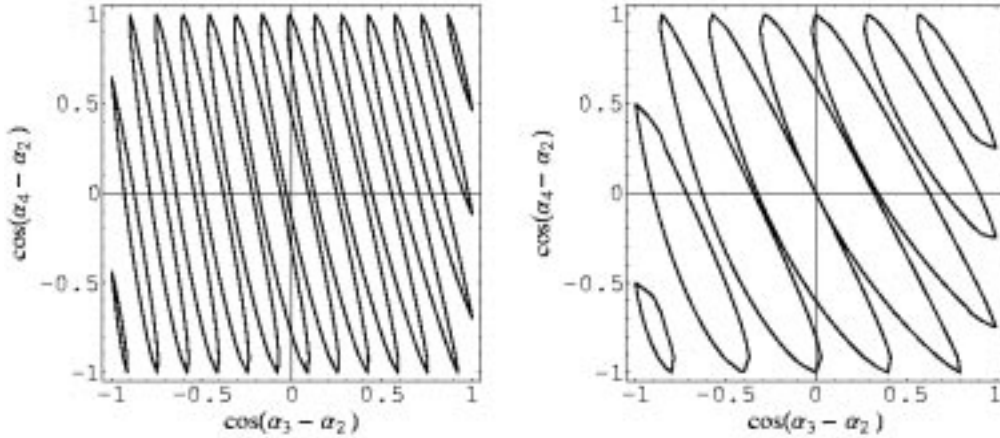


Figure 15: The interdependence of the two CP-violating phases,  $\alpha_{42}$  and  $\alpha_{32}$ , for a given value of  $|\langle m \rangle|$  in the case of the 3+1C neutrino mass spectrum. The figures are obtained for *i*) the maximum allowed value of  $\Delta m_{\text{atm}}^2 = 8.0 \times 10^{-3}$  eV and  $|\langle m \rangle| = \sqrt{0.10 + 0.05n} \times 10^{-1}$  eV with  $n = 0, 1 \dots 15$  (with increasing  $|\langle m \rangle|$  from left to right) (left-hand plot) and *ii*) the minimum allowed value of  $\Delta m_{\text{atm}}^2 = 1.6 \times 10^{-3}$  eV and  $|\langle m \rangle| = \sqrt{3 + 2n} \times 10^{-2}$  eV with  $n = 0, 1 \dots 7$  (with increasing  $|\langle m \rangle|$  from left to right) (right-hand plot) and for  $m_1 = 0$ ,  $|U_{e1}|^2 \leq 0.08$ , the best fit value of  $|U_{e2}|^2$  found in ref. [55] and the maximum allowed value of  $|U_{e4}|^2 \sqrt{\Delta m_{\text{SBL}}^2} = 5.0 \times 10^{-3}$  eV from ref. [35]. The values of  $\cos \alpha_{32,42} = 0, \pm 1$ , correspond to CP-invariance.

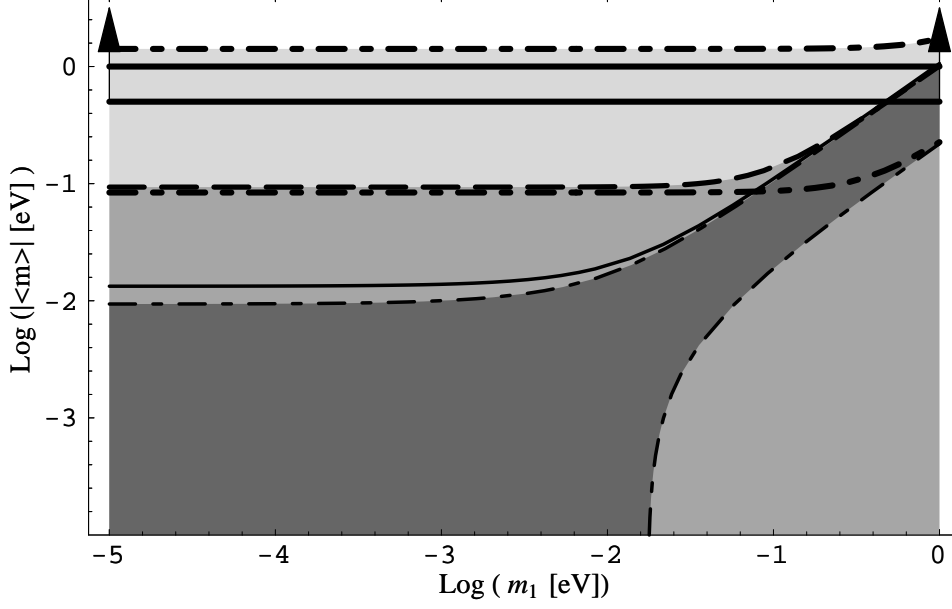


Figure 16: The dependence of  $|\langle m \rangle|$  on  $m_1$  for the LMA solution of the solar- $\nu$  problem. For the 2+2 neutrino mass spectra, the figure is obtained using the allowed values of  $\Delta m_{\text{SBL}}^2$  and  $\theta_{\text{SBL}}$  at 95% C.L. derived in ref. [35], of  $\Delta m_{\text{atm}}^2$  found at 90% C.L. in ref. [41], of  $\Delta m_{\odot}^2$  and  $\theta_{\odot}$  obtained for  $\cos^2 \beta = 0.3$  at 90% C.L. in ref. [42], and the 90% C.L. limit on  $\theta_{\text{CHOOZ}}$  from ref. [58]. For the 3+1 neutrino mass spectra, the allowed regions of  $|\langle m \rangle|$  are derived using the 95% C.L. allowed ranges of  $\Delta m_{\text{SBL}}^2$  and  $\theta_{\text{SBL}}$  from ref. [35] and the allowed values of  $\Delta m_{\text{atm}}^2$ ,  $\Delta m_{\odot}^2$ ,  $\theta_{\odot}$  and  $\theta_{\text{CHOOZ}}$  at 90% C.L. from ref. [55]. The allowed regions for  $|\langle m \rangle|$  correspond *i*) for the 2+2A neutrino mass spectrum, eq. (39) - to light-grey, medium-grey and dark-grey regions between the two doubly-thick dash-dotted lines; *ii*) for the 2+2B neutrino mass spectrum, eq. (58) - to the dark-grey region between the two thick dash-dotted lines; *iii*) for the 3+1A neutrino mass spectrum, eq. (79) - to the grey regions below the upper upper doubly-thick dash-dotted line; *iv*) for the 3+1B neutrino mass spectrum, eq. (102) - to the medium-grey and dark-grey region below the thick solid line; *v*) for the 3+1C neutrino mass spectrum, eq. (123) - to the medium-grey and dark-grey region below the doubly-thick dashed line. The two horizontal lines show the upper limits [29], quoted in eq. (4).

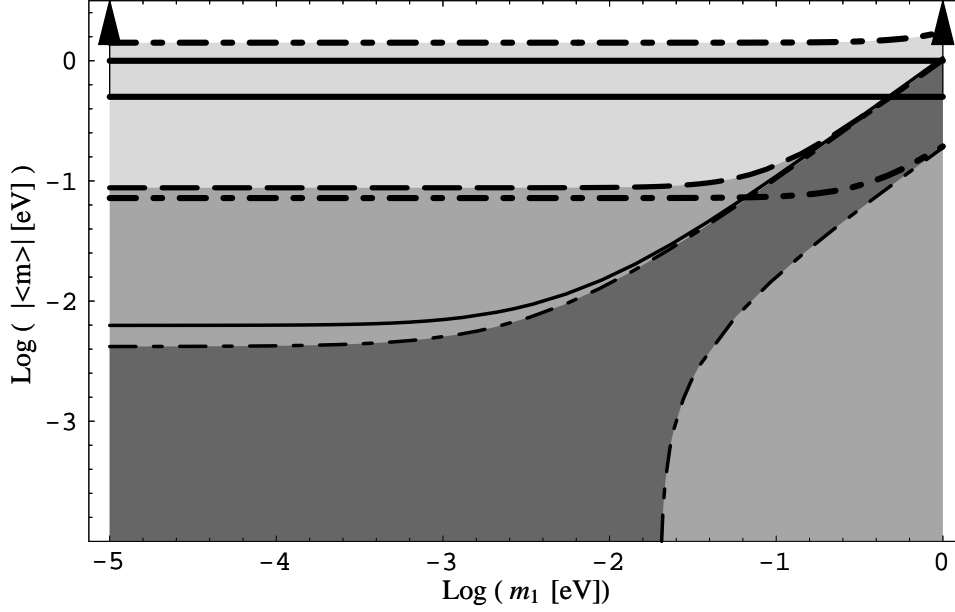


Figure 17: The same as in Fig. 16 but for the LOW-QVO solution of the solar- $\nu$  problem. The allowed values of  $|\langle m \rangle|$  correspond *i*) for the 2+2A neutrino mass spectrum, eq. (39) - to the grey regions between the doubly-thick dash-dotted lines; *ii*) for the 2+2B neutrino mass spectrum, eq. (58) - to the dark-grey region between the thick dash-dotted lines; *iii*) for the 3+1A neutrino mass spectrum, eq. (79) - to the grey regions below the upper doubly-thick dash-dotted line; *iv*) for the 3+1B neutrino mass spectrum, eq. (102) - to the medium-grey and dark-grey region below the thick solid line; *v*) for the 3+1C neutrino mass spectrum, eq. (123) - to the medium-grey and dark-grey region below the doubly-thick dashed line.

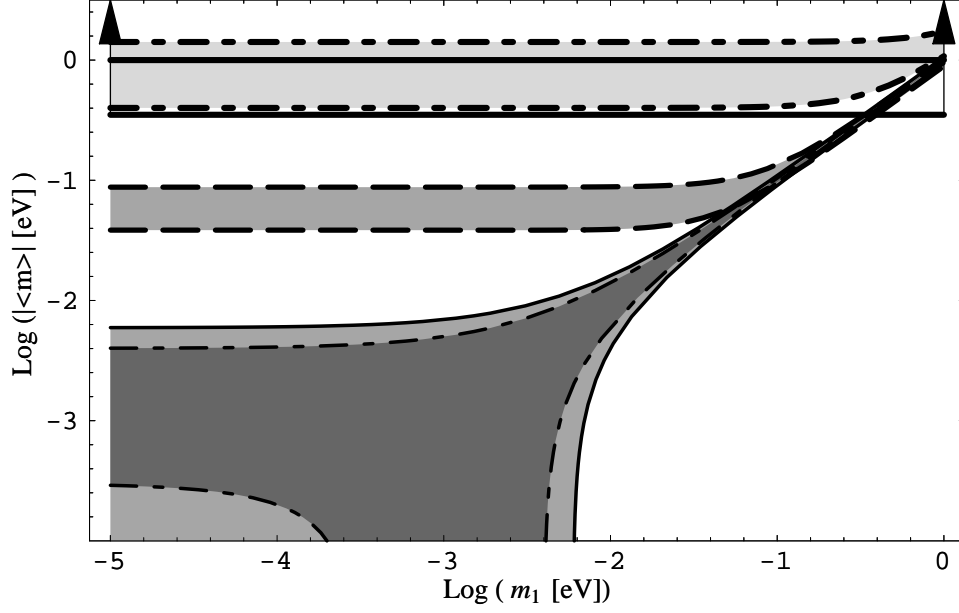


Figure 18: The same as in Fig. 16 but for the SMA MSW solution of the solar neutrino problem. The allowed values of  $\langle m \rangle$  correspond *i*) for the 2+2A and 3+1A neutrino mass spectra, eqs. (39) and (79) - to the light-grey region between the two doubly-thick dash-dotted lines; *ii*) for the 2+2B neutrino mass spectrum, eq. (58) - to the dark grey region between the thick dash-dotted lines and the axes; *iii*) for the 3+1B neutrino mass spectrum, eq. (102) - to the medium-grey and dark-grey region between the two thick solid lines and the axes; *iv*) for the 3+1C neutrino mass spectrum, eq. (123) - to the medium-grey region between the two doubly-thick dashed lines.